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15MAT41

Fourth Semester B.E. Degree Examination, Jan./Feb. 2023
Engineering Mathematics – IV

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1   a. Using Taylor's series method, solve $y' = x + y^2$, $y(0) = 1$ at $x = 0.1$, carryout four decimal places upto third degree terms. **(05 Marks)**

b. Using Runge-Kutta method of fourth order, solve for $y(0.1)$ give that $y' = y(x + y)$, $y(0) = 1$. **(05 Marks)**

c. Given $\dfrac{dy}{dx} = x^2(1+y)$ and $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$, evaluate $y(1.4)$ by Milne's predictor-corrector method. **(06 Marks)**

OR

2   a. Using Euler's modified method to obtain y at $x = 0.2$ given that $\dfrac{dy}{dx} = x + \sqrt{141}$ $y(0) = 1$ and taking $h = 0.2$, correct to four decimal places. **(05 Marks)**

b. Apply Runge-Kutta method, to solve y at $x = 0.2$ given that $\dfrac{dy}{dx} = \dfrac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$ and $h = 0.2$. Carry out the four decimal places. **(05 Marks)**

c. Using Adams-Bashforth method to find $y(0.4)$ given that $y' = x^2 - y$, $y(0) = 1$ and $y(0.1) = 0.9053$, $y(0.2) = 0.8212$, $y(0.3) = 0.7492$. **(06 Marks)**

Module-2

3   a. Prove that $J_{1/2}(x) = \sqrt{\dfrac{2}{\pi x}} \sin x$. **(05 Marks)**

b. Express $x^3 - 5x^2 + 6x + 1$ interms of Legendre's polynomials. **(05 Marks)**

c. Solve $y'' = xy' - y$, $y(0) = 3$, $y'(0) = 0$ to find value of $y(0.1)$, using Runge-Kutta method. **(06 Marks)**

OR

4   a. If $y'' + xy' + y = 0$, $y(0) = 1$, $y'(0) = 0$ given the values $y(0.1) = 0.995$, $y(0.2) = 0.9802$, $y(0.3) = 0.9560$ and $z(0.1) = -0.0995$, $z(0.2) = -0.1960$, $z(0.3) = 0.283$, to find $y(0.4)$, using Milne's predictor-corrector method. **(05 Marks)**

b. Obtain the series solution of Bessel's differential equation. **(05 Marks)**

c. Derive the Rodrigue's formula. **(06 Marks)**

Module-3

5   a. Derive the C-R equation in polarform. **(05 Marks)**

b. Find the bilinear transformation from the points o, i, ∞ onto the points 1, -i, 1 respectively. **(05 Marks)**

c. Discuss the transformation $W = Z + \dfrac{1}{Z}, z \neq 0$. **(06 Marks)**

OR

6 a. State and prove the Cauchy's integral formula. (05 Marks)

   b. Find the analytic function $f(z) = u + iv$, whose real part $u = x^2 - y^2 + \dfrac{x}{x^2 + y^2}$. (05 Marks)

   c. Evaluate $\displaystyle\int_C \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)}\, dz$, where $c : |z| = 3$ using Cauchy's residue theorem. (06 Marks)

Module-4

7 a. The probability density function of a discrete random variable x given table.

x_i	-2	-1	0	1	2	3
$p(x_i)$	0.1	k	0.2	2k	0.3	k

Find K, also calculate mean and variance. (05 Marks)

   b. When a coin is tossed 4 times, find the probability of getting.
       i) Exactly one head ii) At least two heads iii) Atmost two heads. (05 Marks)

   c. If x is an exponential variate with mean 4, evaluate i) $p(0 < x < 1)$ ii) $p(x > 2)$ iii) $p(-\infty < x < 10)$. (06 Marks)

OR

8 a. Derive mean and variance of poisson distribution. (05 Marks)

   b. In a normal distribution 31% of the items are under 45 and 8% are over 64. Find mean and standard deviation of the distribution. (05 Marks)

   c. The joint probability distributions of two random variables x and y as follows:

Y \ -X	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Determine, i) E[X], E[Y] ii) Are x and y are independent random variables? (06 Marks)

Module-5

9 a. A dice was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times on the assumption of random throwing do the data indicate an unbiased dice. [1% level of significance 2.58). (05 Marks)

   b. Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that mean height of the inverse is 66 inches ($t_{0.05} = 2.262$ for q.d.f). (05 Marks)

   c. Find the unique fixed probability vector of the matrix $\begin{bmatrix} 1/4 & 3/4 \\ 5/6 & 1/6 \end{bmatrix}$. (06 Marks)

OR

10 a. Define the following: i) Probability vector ii) Fixed points iii) State space. (05 Marks)

   b. The theory predicts the projections of beans in the four groups G_1, G_2, G_3, G_4 should be in the ratio 9 : 3 : 3 : 1. In experiment with 1600 beans the number in the groups were 882, 313, 287 and 118. Does the experimental result support the theory? (05 Marks)

   c. A students study habits are as follows if the studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night he is 60% sure to study the next night. In the long run how often does he study? (06 Marks)

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