CBCS SCHEME

USN

18MAT21

Second Semester B.E. Degree Examination, Jan./Feb. 2023
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Find the Directional derivative of $\phi = xy^2 + yz^3$ at (2, -1, 1) along i + 2j + 2k. **(06 Marks)**

b. Find div $\vec{A}$, Curl $\vec{A}$, div (Curl $\vec{A}$), where $\vec{A} = x^2yi + y^2zj + z^2yk$. **(07 Marks)**

c. If $u = x^2 yz$, $v = xy - 3z^2$, then find $\nabla.(\nabla u \times \nabla v)$. **(07 Marks)**

OR

2 a. Find the work done in moving a particle in the force field $\vec{F} = 3x^2 i + (2xz - y)j + zk$ along the straight line from (0, 0, 0) to (2, 1, 3). **(06 Marks)**

b. Using Divergence theorem, evaluate $\iiint_V \operatorname{div} \vec{F}\, dv$, where V is the region bounded by the planes x = 0, y = 0, z = 0 and 2x + 2y + z = 4. **(07 Marks)**

c. Using Stoke's theorem, evaluate $\int_C (x + y)dx + (2x - y)dy + (y + z)dz$, where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6). **(07 Marks)**

Module-2

3 a. Solve $(4D^4 - 4D^3 - 23D^2 + 12D + 36)y = 0$. **(06 Marks)**

b. Solve $(D^2 - 4D + 3)y = e^{3x} + 2^x + 7$. **(07 Marks)**

c. Using the method of variation of parameter, solve $(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$. **(07 Marks)**

OR

4 a. Solve the differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = \sin 2x + x$. **(06 Marks)**

b. Solve $(1 - 2x)^2 y'' + 6(1 - 2x)y' + 16y = 4(1-2x)^2$. **(07 Marks)**

c. The current i and the charge q in a series circuit containing an inductance L, capacitance C, e.m.f E satisfy the differential equation $L\frac{di}{dt} + \frac{q}{c} = E$, $i = \frac{dq}{dt}$. Express q in terms of t, given that L, C, E are constants and the value of i, q are both zero initially. **(07 Marks)**

Module-3

5 a. Find the partial differential equation by eliminating the function from $Z = f(x^2 + y^2) + x + y$. **(06 Marks)**

b. Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = 18xy^2 + \sin(2x - y)$. **(07 Marks)**

c. Find all possible solution of $u_{tt} = c^2 u_{xx}$ one dimensional wave equation by Variable Separable method. **(07 Marks)**

OR

6 a. Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} - 3z = 0$, $z = 1$, $\frac{\partial z}{\partial x} = 1$, when $x = 0$. **(06 Marks)**

b. Find the general solution of $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$. **(07 Marks)**

c. Derive one dimensional heat equation. **(07 Marks)**

Module-4

7 a. Test for converges for $\frac{2}{3} + \frac{2.3}{3.5} + \frac{2.3.4}{3.5.7} + \ldots\ldots$ **(06 Marks)**

b. With usual notation prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$. **(07 Marks)**

c. Express $f(x) = x^4 - 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomial. **(07 Marks)**

OR

8 a. Discuss the nature of the series $\frac{2}{3} + \left(\frac{3}{5}\right)^2 + \left(\frac{4}{7}\right)^3 + \ldots\ldots\infty$. **(06 Marks)**

b. Obtain the series solution of Legendre differential equation in terms of $P_n(x)$.
$(1 - x^2)y'' - 2xy' + n(n+1)y = 0$. **(07 Marks)**

c. Show that i) $P_2(\cos\theta) = \frac{1}{4}(1 + 3\cos 2\theta)$ ii) $P_3(\cos\theta) = \frac{1}{8}[3\cos\theta + 5\cos 3\theta]$. **(07 Marks)**

Module-5

9 a. Find the population of a town for the year 1974. Given that

Year	1939	1949	1959	1969	1979	1989
Population in thousands	12	15	20	27	39	52

(06 Marks)

b. Using Newton's general interpolation formula, find the polynomial and hence find f(3).

x	0	1	2	4	5	6
y	22	48	50	30	32	58

(07 Marks)

c. Using Newton Raphson method, find correct to 4(four) decimal places, the smallest root of $\log x = \cos x$. **(07 Marks)**

OR

10 a. Using Regula Falsi method, determine a solution of $2x = \cos x + 3$ correct to four decimal places. **(06 Marks)**

b. Find the polynomial f(x) using Lagrange's Interpolation formula for

x	1	3	4	6
y	0	12	33	135

Hence find f(2). **(07 Marks)**

c. Use Weddle's rule to find $\int_0^{0.6} e^{-x^2}\,dx$, by taking seven ordinates. **(07 Marks)**

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