# CBCS SCHEME

USN | | | | | | | | | |

15CS36



## Third Semester B.E. Degree Examination, Jan./Feb. 2023
## Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

***Note: Answer any FIVE full questions, choosing ONE full question from each module.***



### Module-1

1 a. Define tautology. Prove that, for any propositions p, q, r the compound proposition $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology. **(06 Marks)**

b. Prove the following logical equivalence using laws of logic
$(p \to q) \wedge [\sim q \wedge (r \vee \sim q)] \Leftrightarrow \sim (q \vee p)$ **(05 Marks)**

c. Show that the following argument is valid,

$$\begin{array}{l} p \to q \\ r \to s \\ p \vee r \\ \hline \therefore q \vee s \end{array}$$

**(05 Marks)**

**OR**

2 a. Give (i) A direct proof
(ii) An indirect proof
(iii) Proof by contradiction.
For the statement "If n is an odd integer, then $n+11$ is an even integer". **(06 Marks)**

b. Determine the truth value of each of the following quantified statements, the universe being the set of all non-zero integers.
(i) $\exists x, \exists y[xy = 1]$ (ii) $\exists x, \forall y[xy = 1]$ (iii) $\forall x, \exists y[xy = 1]$
(iv) $\exists x, \exists y, [(2x + y = 5) \wedge (x - 3y = -8)]$ (v) $\exists x, \exists y[(3x - y = 17) \wedge (2x + 4y = 3)]$
**(05 Marks)**

c. Let p(x), q(x) and r(x) be open statements that are defined for a given universe, show that the argument

$$\begin{array}{l} \forall x, [p(x) \to q(x)] \\ \forall x, [q(x) \to r(x)] \\ \hline \therefore \exists x, [p(x) \to r(x)] \end{array}$$

**(05 Marks)**

### Module-2

3 a. If n is any positive integer. Prove that $1.2 + 2.3 + 3.4 + ...... + n(n+1) = \frac{1}{3}n(n+1)(n+2)$ using mathematical induction. **(06 Marks)**

b. How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 50,00,000 **(05 Marks)**

c. A certain question paper contains two parts A and B each containing 4 questions. How many different ways a student can answer 5 questions by selecting at least 2 questions from each part? **(05 Marks)**

**OR**

4 a. Show that $2^n > n^2$ for all positive integers n greater than 4. **(06 Marks)**

b. How many arrangements are there for all letters in the word SOCIOLOGICAL ? How many of these arrangements,

(i) A and G are adjacent?

(ii) All the vowels are adjacent? **(05 Marks)**

c. Determine the co-efficients of,

(i) $xyz^2$ in the expansion of $(2x-y-z)^4$

(ii) $x^2y^2z^3$ in the expansion of $(3x-2y-4z)^7$ **(05 Marks)**

## Module-3

5 a. For any non-empty sets A, B, C prove that

(i) $A\times(B\cup C)=(A\times B)\cup(A\times C)$

(ii) $A\times(B-C)=(A\times B)-(A\times C)$ **(06 Marks)**

b. Prove that, A function $f: A\to B$ is invertible if and only if it is one-to-one and onto. **(05 Marks)**

c. Prove that the relation "Congruent modulo n" is an equivalence relation on the set of all integers z and n > 1. **(05 Marks)**

**OR**

6 a. Let $f: R\to R$ be defined by $f(x)=\begin{cases}3x-5 & \text{for } x>0\\ -3x+1 & \text{for } x\le 0\end{cases}$. Determine f(0), $f\left(\frac{5}{3}\right)$, $f^{-1}(-1)$, $f^{-1}(3)$, $f^{-1}([-5,\ 5])$. **(06 Marks)**

b. Define composition of functions and consider the functions f and g define by $f(x)=x^3$ and $g(x)=x^2+1$, $\forall x\in R$. Find $g\bullet f$, $f\bullet g$, $f^2$, $g^2$ **(05 Marks)**

c. Draw the Hasse diagram representing the positive divisors of 36. **(05 Marks)**

## Module-4

7 a. Determine the number of positive integers n such that $1\le n\le 100$ and n is not divisible by 2, 3 or 5. **(06 Marks)**

b. Find the number of derangements of 1, 2, 3, 4. **(05 Marks)**

c. Solve the recurrence relation $a_n-6a_{n-1}+9a_{n-2}=0$ for $n\ge 2$ given that $a_0=5$, $a_1=12$. **(05 Marks)**

**OR**

8 a. Five teachers $T_1$, $T_2$, $T_3$, $T_4$, $T_5$ to be made class teachers for five classes, $C_1$, $C_2$, $C_3$ $C_4$, $C_5$ one teacher for each class. $T_1$ and $T_2$ do not wish to become the class teachers for $C_1$ or $C_2$, $T_3$ and $T_4$ for $C_4$ or $C_5$, and $T_5$ for $C_3$ or $C_4$ or $C_5$. In how many ways can the teachers be assigned the work (without displeasing any teachers)? **(06 Marks)**

b. In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? **(05 Marks)**

c. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use recurrence relation to determine the number of virus affected files in the system after one day. **(05 Marks)**

## Module-5

9 a. Explain Konigsberg Bridge problem with figure. (06 Marks)

b. Show that the following graphs are isomorphic

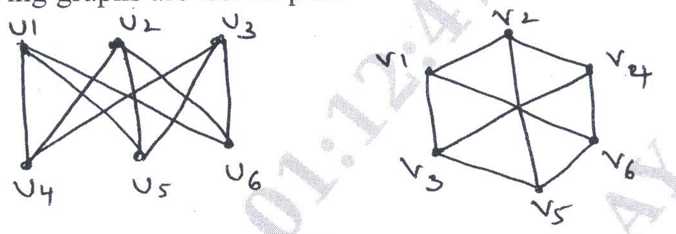


Fig Q9(b) (05 Marks)

c. Using the Merge – sort method, sort the list 7, 3, 8, 4, 5, 10, 6, 2, 9. (05 Marks)

**OR**

10 a. Obtain an optical prefix code for the message ROAD IS GOOD. Indicate the code. (06 Marks)

b. Define: i) Regular graph ii) Complete graph iii) Bipartite graph
iv) Subgraph v) Complete bipartite graph. (05 Marks)

c. In every tree T = (V, E) prove that |V| = |E| + 1. (05 Marks)

* * * * *