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17MATDIP41

Fourth Semester B.E. Degree Examination, July/August 2022
Additional Mathematics – II

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the rank of a matrix $A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$ by reducing to echelon form. **(07 Marks)**

   b. Use Cayley-Hamilton theorem to find the inverse of a matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$. **(07 Marks)**

   c. Solve the following system of equation of Gauss Elimination method:
   $x + y + z = 9$
   $x - 2y + 3z = 8$
   $2x + y - z = 3$. **(06 Marks)**

OR

2 a. Test for consistency and solve
   $5x_1 + x_2 + 3x_3 = 20$
   $2x_1 + 5x_2 + 2x_3 = 18$
   $3x_1 + 2x_2 + x_3 = 14$. **(07 Marks)**

   b. Find all the Eigenvalues of the matrix
   $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. **(07 Marks)**

   c. Find the rank of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ -1 & 2 & 3 \\ 1 & 5 & 7 \end{bmatrix}$. **(06 Marks)**

Module-2

3 a. Solve $\dfrac{d^3y}{dx^3} - 2\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} - 8y = 0$. **(07 Marks)**

   b. Solve $y'' - 4y' + 13y = \cos 2x$. **(07 Marks)**

   c. Solve $\dfrac{d^3y}{dx^3} + 2\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = x^3$. **(06 Marks)**

OR

4 a. Solve by the method of variation of parameters, $y'' - 2y' + y = e^x . \log x$. **(07 Marks)**

   b. Solve by the method of undetermined coefficients $(D^2 + 1)y = \sin x$. **(07 Marks)**

   c. Solve $\dfrac{d^2y}{dx^2} - 4y = 3^x$. **(06 Marks)**

Module-3

5 a. Find the Laplace transform of $\cos t \times \cos 2t . \cos 3t$. (07 Marks)
   b. Find the Laplace transform of $e^{3t} \sin 5t . \sin 3t$. (07 Marks)
   c. Find the Laplace transform of $t^3 \sin t$. (06 Marks)

OR

6 a. If f(t) is a periodic function of period T > 0, then prove that $L\{f(t)\} = \dfrac{1}{1-e^{-sT}} \displaystyle\int_0^T e^{-st} f(t) dt$. (07 Marks)

   b. Find the Laplace transform of $f(t) = E \sin wt$, $0 < t < \pi/w$ having period π/w. (07 Marks)

   c. Express $f(t) = \begin{cases} \cos t & 0 < t < \pi \\ \cos 2t & \pi < t < 2\pi \\ \cos 3t & t > 2\pi \end{cases}$ as a unit step function and hence find its Laplace transform. (06 Marks)

Module-4

7 a. Find the Laplace of $\dfrac{1}{(s-1)(s+1)(s+2)}$. (07 Marks)

   b. Solve $y''' + 2y'' - y' - 2y = 0$ given $y(0) = y'(0) = 0$ and $y''(0) = 6$ by using Laplace transform. (07 Marks)

   c. Find: $L^{-1}\left[\dfrac{3s+2}{(s-2)(s+1)}\right]$. (06 Marks)

OR

8 a. Find $L^{-1}[\cot^{-1}(s/a)]$. (07 Marks)

   b. Employ Laplace transform to solve the equation $y'' + 5y' + 6y = 5e^{2x}$, $y(0) = 2$, $y'(0) = 1$. (07 Marks)

   c. Find the inverse Laplace transform of $\log\left[\dfrac{s+4}{s-4}\right]$. (06 Marks)

Module-5

9 a. State and prove Bayes theorem. (07 Marks)
   b. Prove that
      $P(A \cup B \cup C) = P(A) + P(B) + P(C) + P(A \cap B \cap C) - P(A \cap B) - P(B \cap C) - P(C \cap A)$. (07 Marks)
   c. A pair of dice is tossed twice. Find the probability of scoring 7 points
      i) Once   ii) atlesat once   iii) twice. (06 Marks)

OR

10 a. If A and B are two events having P(A) = 1/2, P(B) = 1/3 and P(A∩B) = 1/4 compute
       i) P(A/B)   ii) P(B/A)   iii) $P(\overline{A}/\overline{B})$. (07 Marks)
    b. Three machines A, B and C produce respectively 60%, 30%, 10% of the total number of items of a factory. The percentage of defective output of these machines are respectively 2%, 3% and 4%. An item is selected at random and is found defective. Find the probability that the item was produced by machine C. (07 Marks)
    c. In a school 25% of the students failed in first language, 15% of the students failed in second language and 10% of the students failed in both. If a student is selected at random find the probability that.
       i)   He failed in first language if he had failed in the second language.
       ii)   He failed in second language if he had failed in the first language.
       iii) He failed in either of the two languages. (06 Marks)

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