USN

MATDIP401

Fourth Semester B.E. Degree Examination, July/August 2022
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Prove that the sum of the squares of the direction cosines in equal to unity. **(06 Marks)**

b. If $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are the direction cosines of a line. Prove that

(i) $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$.

(ii) $\cos 2\alpha+\cos 2\beta+\cos 2\gamma=-1$ **(07 Marks)**

c. Find the image of the point (2, −1, 3) in the plane $2x+4y+z-24=0$. **(07 Marks)**

2 a. Find the equation of the plane in the intercept form. **(06 Marks)**

b. Find the equation of the plane which passes through (3, −3, 1) and is perpendicular to the planes $7x+y+2z=6$ and $3x+5y-6z=8$. **(07 Marks)**

c. Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ are coplanar. Find their common point. **(07 Marks)**

3 a. Find sine of the angle between the vectors $2\hat{i}-2\hat{j}+\hat{k}$ and $\hat{i}-2\hat{j}+2\hat{k}$. **(06 Marks)**

b. Find the constant 'a' such that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+a\hat{j}+5\hat{k}$ are coplanar. **(07 Marks)**

c. Prove that $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right]=\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$. **(07 Marks)**

4 a. A particle moves along the curve $x=1-t^3$, $y=1+t^2$, $z=2t-5$ where t is the time. Find the velocity and acceleration at t = 1. **(06 Marks)**

b. Find the unit normal vector to the surface $xy+x+zx=3$ at (1, 1, 1). **(07 Marks)**

c. Find the angle between the surfaces $x^2+y^2+z^2=9$ and $x=z^2+y^2-3$ at the point (2, -1, 2). **(07 Marks)**

5 a. Find the directional derivative of $\phi=x^2yz+xz^2$ at the point $(-1,2,1)$ in the direction of $2\hat{i}-\hat{j}-2\hat{k}$. **(06 Marks)**

b. Show that the vectors $\vec{F}=(2xy+z^2)i+(x^2+2xy)j+(y^2+2zx)k$ is irrotational. **(07 Marks)**

c. Given that $\vec{F}=(x+y+1)\hat{i}+\hat{j}-(x+y)\hat{k}$, show that $\vec{F}.\text{curl}\vec{F}=0$. **(07 Marks)**

6 a. Using the definition show that $L[t^n]=\frac{n!}{s^{n+1}}$. **(05 Marks)**

b. Find $L[t\cos at]$. **(05 Marks)**

c. Find $L\left[\frac{\cos at-\cos bt}{t}\right]$. **(05 Marks)**

d. Find $L[\cos(at+b)]$. **(05 Marks)**

7 a. Find $L^{-1}\left[\dfrac{s^2-3s+4}{s^3}\right]$. **(05 Marks)**

b. Find $L^{-1}\left[\dfrac{s+2}{s^2-4s+13}\right]$. **(05 Marks)**

c. Find $L^{-1}\left[\dfrac{s^2+s-2}{s(s+3)(s-2)}\right]$. **(05 Marks)**

d. Find $L^{-1}\left[\log\dfrac{(s+a)}{(s+b)}\right]$. **(05 Marks)**

8 a. Using Laplace Transform method solve $y''+2y'-3y=\sin t$ subject to the condition, $y(0)=y'(0)=0$. **(10 Marks)**

b. By applying Laplace transform, solve the differential equation $y''+4y'+3y=0$ subject to the condition $y(0)=0$ and $y'(0)=1$. **(10 Marks)**

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