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MATDIP301

Third Semester B.E. Degree Examination, July/August 2022
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the modulus and amplitude of $\frac{1}{1-\cos\alpha + i\sin\alpha}$. **(07 Marks)**

b. Express $\frac{(3+i)(1-3i)}{2+i}$ in the form of x + iy. **(06 Marks)**

c. Find the cube roots of 1-i **(07 Marks)**

2 a. Find the n^{th} derivative of $\frac{x}{(x+1)(2x+1)}$. **(07 Marks)**

b. Find the n^{th} derivative of cosx cos3x cos5x. **(06 Marks)**

c. If $y = a\cos(\log x) + b\sin(\log x)$ show that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(07 Marks)**

3 a. Show that the following pairs of curves intersect orthogonally $r_1 = a(1+\sin\theta)$ and $r_2 = a(1-\sin\theta)$. **(07 Marks)**

b. With usual notations prove that $\tan\phi = r\frac{d\theta}{dr}$. **(06 Marks)**

c. Expand $\sqrt{1+\sin 2x}$ by Maclaurin's series upto the term containing x^4. **(07 Marks)**

4 a. State and prove Euler's theorem on homogeneous functions. **(07 Marks)**

b. If $u = f(x-y, y-z, z-x)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. **(06 Marks)**

c. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, -1, 0). **(07 Marks)**

5 a. Obtain a reduction formula for $I_n = \int_0^{\pi/2} \cos^n x\, dx$, n being a positive integer. **(07 Marks)**

b. Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)\, dy\, dx$. **(06 Marks)**

c. Evaluate $\int_{-1}^{1} \int_0^z \int_{x-z}^{x+z} (x+y+z)\, dy\, dx\, dz$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

6 a. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. **(07 Marks)**

b. Express $I = \int_0^1 x^{3/2}(1-x)^{1/2}\,dx$ in terms of beta function. **(06 Marks)**

c. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\,d\theta = \pi$. **(07 Marks)**

7 a. Solve: $(2x+y+1)\,dx + (x+2y+1)\,dy = 0$. **(07 Marks)**

b. Solve $\frac{dy}{dx} + y\cot x = \cos x$. **(06 Marks)**

c. Solve: $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$. **(07 Marks)**

8 a. Solve: $6\frac{d^2y}{dx^2} + 17\frac{dy}{dx} + 12y = e^{-x}$. **(07 Marks)**

b. Solve: $4\frac{d^4y}{dx^4} - 8\frac{d^3y}{dx^3} - 7\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$. **(06 Marks)**

c. Solve: $\frac{d^3y}{dx^3} - y = 3\cos 2x$. **(07 Marks)**

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