BRIDGE COURSE

USN | | | | | | | | | | MATDIP301

Third Semester B.E. Degree Examination, July/August 2022
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the modulus and amplitude of the complex number $\frac{4+2i}{2-3i}$. **(06 Marks)**

b. Express the complex number $\frac{(1-i)(2-i)}{3-i}$ in the form $x+iy$. **(06 Marks)**

c. (i) Define complex number
(ii) If $(3x-2iy)(2+i)=10(1+i)$, then find the values of x and y. **(08 Marks)**

2 a. Find the n^{th} derivative of e^{ax}. **(06 Marks)**

b. Find the n^{th} derivative of $\cos^2 3x$. **(06 Marks)**

c. If $y=\tan^{-1}x$, prove that $(1+x^2)y_{n+2}+2(n+1)xy_{n+1}+n(n+1)y_n=0$. **(08 Marks)**

3 a. Find the angle between the radius vector and the tangent to the curve $r=a(1-\cos\theta)$. **(06 Marks)**

b. Obtain the Maclaurin's series expansion of e^x, upto the term containing x^4. **(06 Marks)**

c. With usual notations, prove that (i) $p=r\sin\phi$ (ii) $\frac{1}{p^2}=\frac{1}{r^2}+\frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$. **(08 Marks)**

4 a. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$, when $f=\log(x^2+y^2)$. **(06 Marks)**

b. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$. **(06 Marks)**

c. If $u=x^2-y^2$, $v=y^2$, find the value of $\frac{\partial(u,v)}{\partial(x,y)}$. **(08 Marks)**

5 a. Evaluate $\int_{y=0}^{y=1}\int_{x=0}^{x=6} xy\,dx\,dy$. **(06 Marks)**

b. Evaluate $\int_0^{\pi}\cos^5\left(\frac{x}{2}\right)dx$. **(06 Marks)**

c. Obtain the reduction formula for $\int \sin^n x\,dx$, where n is a positive integer. **(08 Marks)**

6 a. Evaluate $\iint_R xy\,dx\,dy$, where R is the region in positive quadrant for which $x + y \leq 1$. **(06 Marks)**

b. Evaluate $\int_{z=-1}^{z=1}\int_{y=-2}^{y=2}\int_{x=-3}^{3} dx\,dy\,dz$. **(06 Marks)**

c. (i) Define Gamma function.

(ii) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. **(08 Marks)**

7 a. Solve : $\frac{dy}{dx} = \frac{\cos^2 y}{\cos^2 x}$. **(06 Marks)**

b. Solve : $(y^3 - 3x^2y)dx + (3xy^2 - x^3)dy = 0$. **(06 Marks)**

c. Solve : $\left(y\left(1+\frac{1}{x}\right)+\cos y\right)dx + (x + \log x - x\sin y)dy = 0$. **(08 Marks)**

8 a. Solve : $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$. **(06 Marks)**

b. Solve : $(D^2 + 6D + 9)y = 0$. **(06 Marks)**

c. Solve : $(D^3 - 7D - 6)y = 0$. **(08 Marks)**

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