CBCS SCHEME

USN

21MAT21

# Second Semester B.E. Degree Examination, July/August 2022
## Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

***Note: Answer any FIVE full questions, choosing ONE full question from each module.***





**Module-1**

1 a. Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dxdydz$. **(06 Marks)**

b. Evaluate $\int_{0}^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy\,dydx$ by changing the order of integration. **(07 Marks)**

c. Prove that $\beta(m,n)=\dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

**OR**

2 a. Evaluate $\int_{0}^{\infty}\int_{0}^{\infty} e^{-(x^2+y^2)}\,dxdy$ by changing to polar coordinates. **(06 Marks)**

b. Find the area between the parabolas $y^2=4ax$ and $x^2=4ay$. **(07 Marks)**

c. Prove that $\int_{0}^{\frac{\pi}{2}}\sqrt{\cot\theta}\,d\theta=\dfrac{\pi}{\sqrt{2}}$. **(07 Marks)**

**Module-2**

3 a. Find the directional derivative of $\phi=\dfrac{xz}{x^2+y^2}$ at the point (1, -1, 1) in the direction of $\hat{i}-2\hat{j}+\hat{k}$. **(06 Marks)**

b. Find $\operatorname{div}\vec{F}$ and $\operatorname{curl}\vec{F}$, where $\vec{F}=\operatorname{grad}(xy^3z^3)$. **(07 Marks)**

c. If $\vec{F}=(x+y+az)\hat{i}+(bx+2y-z)\hat{j}+(x+cy+2z)\hat{k}$, find a, b, c such that $\vec{F}$ is irrotational. **(07 Marks)**

**OR**

4 a. If $\vec{F}=xy\hat{i}+(x^2+y^2)\hat{j}$, evaluate $\int_C \vec{F}\cdot\vec{dr}$ along the curve $C: y=x^2-4$ in the xy-plane from the point (2, 0) to (4, 12). **(06 Marks)**

b. Using Green's theorem, evaluate $\int(y-\sin x)dx+\cos x\,dy$ where C is the triangle in the xy-plane bounded by the lines $y=0$, $x=\dfrac{\pi}{2}$ and $y=\dfrac{2x}{\pi}$. **(07 Marks)**

c. Using Stokes theorem, evaluate $\oint_C \vec{F}.\vec{dr}$, where $\vec{F}=(x^2+y^2)\hat{i}-2xy\hat{j}$ taken around the rectangle bounded by x = 0, x = a, y = 0, y = b. **(07 Marks)**

## Module-3

5 a. Form the partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2)$ (06 Marks)

b. Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that x = 0, z = 0 and $\frac{\partial z}{\partial x} = a \sin y$. (07 Marks)

c. Derive one dimensional wave equation, $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$. (07 Marks)

**OR**

6 a. Form the partial differential equation by eliminating the arbitrary function from, $x+y+z = f(x^2+y^2+z^2)$ (06 Marks)

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$, when x= 0 and z = 0 when y is an odd multiple of $\frac{\pi}{2}$. (07 Marks)

c. Solve $(x + 2z)p + (4zx - y)q = (2x^2 + y)$ (07 Marks)

## Module-4

7 a. Find a root of the equation $\tan x = x$ which is near to x = 4.5 using Newton's Raphson method. (06 Marks)

b. Given $\sin 45^\circ = 0.7071$, $\sin 50^\circ = 0.7660$, $\sin 55^\circ = 0.8192$, $\sin 60^\circ = 0.8660$ find $\sin 52^\circ$ using Newton's forward interpolation formula. (07 Marks)

c. Evaluate $\int_0^1 \sqrt{\sin x + \cos x}\, dx$ correct to two decimal places using Simpson's $\frac{1}{3}^{rd}$ rule taking seven Equi distance ordinates. (07 Marks)

**OR**

8 a. Find the root of the equation $x \log_{10} x = 1.2$ that lies between 2 and 3 correct to three decimal places, using Regula Falsi method. (06 Marks)

b. Using Newton's divided difference formula find f(4) given that,

| | | | | |
|---|---|---|---|---|
| x | 0 | 2 | 3 | 6 |
| f(x) | –4 | 2 | 14 | 158 |

(07 Marks)

c. Evaluate $\int_0^{0.3} \sqrt{1 - 8x^3}\, dx$ using Simpson's $\left(\frac{3}{8}\right)^{th}$ rule by taking seven ordinates. (07 Marks)

## Module-5

9 a. Solve $\frac{dy}{dx} = e^x - y$, $y(0) = 2$ using Taylor's series method upto $4^{th}$ degree terms at any point x. (06 Marks)

b. Using modified Euler's method, find y at x = 0.2 from $\frac{dy}{dx} = 3x + \frac{y}{2}$ with y(0) = 1 taking h = 0.1 perform two iteration at each step. (07 Marks)

c. Solve $\frac{dy}{dx} = 2e^x - y$ given that $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$, $y(0.3) = 2.090$ find y(0.4) using Milne's predictor corrector method. (07 Marks)

**OR**

10 a. Employ Taylor's series method to obtain the value of y at x = 0.1 for the equation $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$ considering upto $4^{th}$ degree term. **(06 Marks)**

b. Use Runge Kutta method of order 4 find y at x = 0.2 given that $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, y(0) = 1 taking h = 0.2. **(07 Marks)**

c. Apply Milne's predictor corrector method to find y(1.4) from $\frac{dy}{dx} = x^2 + \frac{y}{2}$ given that $y(1) = 2$, y(1.1) = 2.2156, y(1.2) = 2.4549, y(1.3) = 2.7514. **(07 Marks)**

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