CBCS SCHEME

USN

18MAT21

Second Semester B.E. Degree Examination, July/August 2022
Advanced Calculus and Numerical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point (1, -2, -1) in the direction of the vector $2i - j - 2k$. **(06 Marks)**

b. Find $\text{div}\,\vec{F}$ and $\text{curl}\,\vec{F}$ where $\vec{F} = \text{Grad}(x^3 + y^3 + z^3 - 3xyz)$. **(07 Marks)**

c. If $\vec{F} = 3x^2i + (2xz - y)j + zk$ find the work done in moving a particle along the curve, $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$. **(07 Marks)**

OR

2 a. Find the values of a, b, c such that $\vec{F} = (axy + bz^3)i + (3x^2 - cz)j + (3xz^2 - y)k$ is a conservative force field. Hence find the scalar potential ϕ such that $\vec{F} = \nabla\phi$. **(06 Marks)**

b. Using Green's theorem evaluate,
$\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region enclosed by $y = \sqrt{x}$ and $y = x^2$. **(07 Marks)**

c. Using Gauss divergence theorem evaluate $\iint_S \vec{F}.\hat{n}\,ds$ over the rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$ given that $\vec{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$. **(07 Marks)**

Module-2

3 a. Solve $(D-2)^2 y = 8(e^{2x} + \sin 2x)$. **(06 Marks)**

b. Solve $(D^2 + a^2)y = \sec ax$ by the method of variation of parameters. **(07 Marks)**

c. Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$ **(07 Marks)**

OR

4 a. Solve $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$. **(06 Marks)**

b. Solve $(D^2 + 4)y = x^2 + e^{-x}$. **(07 Marks)**

c. Solve $(x+1)^2 \frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} + y = 2\sin(\log(x+1))$. **(07 Marks)**

Module-3

5 a. Form the partial differential equation by eliminating the arbitrary function from $\phi(x+y+z, x^2+y^2-z^2)=0$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2\sin y$ when $x = 0$ and $z = 0$ when y is an odd multiple of $\frac{\pi}{2}$. **(07 Marks)**

c. Derive one dimensional heat equation, $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$. **(07 Marks)**

OR

6 a. Form the partial differential equation by eliminating the arbitrary function from the equation, $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. **(06 Marks)**

b. Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that $x = 0$, $z = e^y$, $\frac{\partial z}{\partial x} = 1$. **(07 Marks)**

c. Find all the possible solutions of one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$ using the method of separation of variables. **(07 Marks)**

Module-4

7 a. Test for convergence of the series,

$$\sum_{n=1}^{\infty} \frac{3.6.9.......3n}{4.7.10.......(3n-1)} \cdot \frac{5^n}{(3n+2)}.$$ **(06 Marks)**

b. With usual notation prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. **(07 Marks)**

c. Express $2x^3 - x^2 - 3x + 2$ in terms of Legendre's polynomial. **(07 Marks)**

OR

8 a. Discuss the convergence of the series,

$$\left(\frac{3}{4}\right)x + \left(\frac{4}{5}\right)^2 x^2 + \left(\frac{5}{6}\right)^3 x^3 + ...$$ **(06 Marks)**

b. If α and β are two roots of $J_n(x) = 0$ then prove that $\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = 0$ if $\alpha \neq \beta$. **(07 Marks)**

c. Express $x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomial. **(07 Marks)**

Module-5

9 a. Using Newton's forward difference formula find f(3) given that,

x	0	2	4	6	8	10
f(x)	0	4	56	204	496	980

(06 Marks)

b. Using Regula-Falsi method find the root of the equation, $xe^x = \cos x$ that lies between 0.4 and 0.6. Carryout 4 iterations. **(07 Marks)**

c. Use Weddle's rule to evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cos\theta}\, d\theta$ on dividing the interval $\left[0, \frac{\pi}{2}\right]$ into 6 equal parts. **(07 Marks)**

OR

10 a. Use Newton Raphson method to find a real root of the equation $x\sin x + \cos x = 0$ near $x = \pi$. Carryout iterations upto 4 decimal places of accuracy. **(06 Marks)**

b. If $y(0) = -12$, $y(1) = 0$, $y(3) = 6$, $y(4) = 12$ find Lagrange's interpolating polynomial and estimate y at x = 2. **(07 Marks)**

c. Using Simpson's $\frac{1}{3}^{rd}$ rule evaluate $\int_0^1 \frac{dx}{1+x^2}$ by taking $h = \frac{1}{6}$. **(07 Marks)**

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