CBCS SCHEME

USN |  |  |  |  |  |  |  |  |  |  | **18MAT11**

## First Semester B.E. Degree Examination, July/August 2022
## Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

*Note: Answer any FIVE full questions, choosing ONE full question from each module.*

### Module-1

1 a. With usual notations prove that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$. **(07 Marks)**

b. Find the angle of intersection of the curves
$r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$. **(06 Marks)**

c. Find the radius of curvature at any point on the curve $y^2 = \frac{a^2(a-x)}{x}$. Where the curve meets x-axis. **(07 Marks)**

**OR**

2 a. Show that the pair of curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersect orthogonally. **(06 Marks)**

b. Find the pedal equation of the curve $r^m \cos m\theta = a^m$. **(06 Marks)**

c. Show that the evolute of the parabola $y^2 = 4ax$ is $27ay^2 = 4(x-2a)^3$. **(08 Marks)**

### Module-2

3 a. Find the Maclaurin's series of Log (secx) upto the terms containing $x^4$. **(07 Marks)**

b. Evaluate $\underset{x \to 0}{\text{Lt}}\left[\frac{a^x + b^x + c^x + d^x}{4}\right]^{1/x}$. **(06 Marks)**

c. Find the extreme values of $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$. **(07 Marks)**

**OR**

4 a. If $u = f(r, s, t)$ where $r = \frac{x}{y}$, $s = \frac{y}{z}$, $t = \frac{z}{x}$ prove that $x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y} + z.\frac{\partial u}{\partial z} = 0$. **(07 Marks)**

b. If x, y, z are the angles of a triangle, find the maximum values of cosxcosycosz. **(07 Marks)**

c. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$ find $J\left(\frac{uvw}{xyz}\right)$. **(06 Marks)**

### Module-3

5 a. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dxdydz$. **(07 Marks)**

b. Evaluate $\int_{0}^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$ by changing the order of integration. **(06 Marks)**

c. Prove that $\beta(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

**OR**

6 a. Evaluate $\iint_R xy\,dx\,dy$ over the positive quadrant of the circle $x^2 + y^2 = 4$. **(07 Marks)**

b. Find the volume of the region bounded by $z = x^2 + y^2$, $z = 0$, $x = -a$, $x = a$ and $y = -a$, $y = a$. **(06 Marks)**

c. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \pi$. **(07 Marks)**

**Module-4**

7 a. Solve $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)\,dy = 0$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + y\tan x = y^2 \sec x$. **(07 Marks)**

c. A body in air at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C what will be the temperature of the body after 40 min. **(07 Marks)**

**OR**

8 a. Solve $(5x^3 + 12x^2 + 6y^2)dx + 6xy\,dy = 0$. **(07 Marks)**

b. Solve $xp^2 - yp + a = 0$. Also find its singular solution. **(06 Marks)**

c. Find the orthogonal trajectories of the family of curves $r = a(1-\cos\theta)$. **(07 Marks)**

**Module-5**

9 a. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ using elementary row transformations. **(06 Marks)**

b. Find the largest eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ with initial vector $[1 \quad 0 \quad 0]^l$. Carry out 6 iterations. **(07 Marks)**

c. Solve the following system of equations by Gauss elimination method.
$2x - 3y + z = 9$, $x + y + z = 6$, $x - y + z = 2$. **(07 Marks)**

**OR**

10 a. Apply Gauss Jordan method to solve the system of equations.
$2x + y + z = 10$, $3x + 2y + 3z = 18$, $x + 4y + 9z = 16$. **(06 Marks)**

b. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ into diagonal form. **(07 Marks)**

c. Solve the following system of equations by Gauss-Seidal method:
$20x + 2y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$ carry out 5 iterations. **(07 Marks)**

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