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First Semester B.E. Degree Examination, July/August 2022
Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the n^{th} derivative of i) $x^2 e^{5x}$ ii) $\cos 2x \cos x$. (06 Marks)
- b. Find the angle between the curves $r = a\theta$ and $r = \frac{a}{\theta}$. (07 Marks)
- c. Find the pedal equation of the curve $r = a(1 + \cos\theta)$. (07 Marks)

OR

- 2 a. If $x = \tan(\log y)$ prove that $(1 + x^2)y_{n+1} + (2nx - 1)y_n + n(n-1)y_{n-1} = 0$. (06 Marks)
- b. Prove with usual notation $\tan \phi = r \left(\frac{d\theta}{dr} \right)$. (07 Marks)
- c. Find the radius of curvature of the curve $y = 4 \sin x - \sin 2x$ at $x = \pi/2$. (07 Marks)

Module-2

- 3 a. Expand $2x^3 + 7x^2 + x - 6$ in power of $(x - 2)$ using Taylor's theorem. (06 Marks)
- b. Evaluate: i) $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$ ii) $\lim_{x \rightarrow 0} (x^x)$ (07 Marks)
- c. If $u = \frac{2yz}{x}$; $v = \frac{3zx}{y}$; $w = \frac{4xy}{z}$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (07 Marks)

OR

- 4 a. If $u = \sin^{-1} \left(\frac{x^2 y^2}{x + y} \right)$ prove that $xu_x + yu_y = 3 \tan u$. (06 Marks)
- b. Using Maclaurin's series prove that $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$. (07 Marks)
- c. Find $\frac{dU}{dt}$, if $u = x^3 y^2 + x^2 y^3$ where $x = at^2$, $y = 2at$ using partial derivatives. (07 Marks)

Module-3

- 5 a. A particle moves along the curve $C: x = t^3 - 4t, y = t^2 + 4t, z = 8t^2 - 3t^3$ where 't' denotes time. Find the components of its acceleration at $t = 2$ along the tangent. (06 Marks)
- b. Find the value of "a" such that $\vec{F} = (axy - z^3)\hat{i} + (a-2)x^2\hat{j} + (1-a)xz^2\hat{k}$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. (07 Marks)
- c. Prove that $\text{div}(\text{curl } \vec{A}) = 0$. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and/or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

- 6 a. Find the divergence and curl of vector $\vec{F} = (xyz)\hat{i} + (3x^2y)\hat{j} + (xz^2 - y^2z)\hat{k}$ at $(2, -1, 1)$. (06 Marks)
- b. Find the directional derivative of $\vec{F} = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$. (07 Marks)
- c. Prove that $\text{curl}(\text{grad } \phi) = \vec{0}$. (07 Marks)

Module-4

- 7 a. Obtain the reduction formula for $\int \sin^n x dx$. (06 Marks)
- b. Solve: $x \frac{dy}{dx} + y(\log y) = xy e^x$. (07 Marks)
- c. Water at temperature 30°C takes 5 minutes to warm upto 50°C in a room temperature of 60°C . Find the temperature after 20 minutes. (07 Marks)

OR

- 8 a. Evaluate $\int_0^\infty \frac{x^2}{(1+x^2)^{7/2}} dx$. (06 Marks)
- b. Solve: $(2xy + e^y)dx + (x^2 + xe^y)dy = 0$. (07 Marks)
- c. Find the orthogonal trajectory of the family of curve $r^2 = a \sin 2\theta$. (07 Marks)

Module-5

- 9 a. Solve the system of equations by Gauss Seidal method.
 $30x - 2y + 3z = 75$; $2x + 2y + 18z = 30$; $x + 17y - 2z = 48$, carry out three iteration. (06 Marks)
- b. Diagonalise the matrix $A = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$. (07 Marks)
- c. Using Rayleigh's power method, find largest Eigen value and corresponding Eigen vector of the matrix $\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ choosing $X_0 = [1 \ 0 \ 0]^T$. (07 Marks)

OR

- 10 a. Find the Rank of the matrix $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$. (06 Marks)
- b. Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into canonical form, using orthogonal transformation. (07 Marks)
- c. Apply Gauss-Jordan method to find the solution of system of equations
 $x + 2y + z = 3$, $3x - y + 2z = 13$, $2x + 3y + 3z = 10$. (07 Marks)
