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MATDIP401

Fourth Semester B.E. Degree Examination, Feb./Mar. 2022
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. If l, m, n are the direction cosines of a straight line, then prove that $l^2 + m^2 + n^2 = 1$. **(06 Marks)**

b. A line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$. **(07 Marks)**

c. With the usual notation, derive the equation of the plane in the form $lx + my + nz = p$. **(07 Marks)**

2 a. Find the equation of plane which passes through (-10, 5, 4) and is normal to the line joining the points (4, -1, 2) and (-3, 2, 3) line joining the points (4, -1, 2) and (-3, 2, 3). **(06 Marks)**

b. Find the image of the point (1, 3, 4) in the plane $2x - y + z + 3 = 0$. **(07 Marks)**

c. Find the equation of a line which passes through the point (-2, 3, 4) and parallel to the planes $2x + 3y + 4z = 5$ and $4x + 3y + 5z = 6$. **(07 Marks)**

3 a. Find the unit normal to both the vectors $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ if $\vec{a} = \hat{i}+\hat{j}-\hat{k}$, $\vec{b} = -\hat{i}+2\hat{y}+\hat{k}$ and $\vec{c} = \hat{i}-2\hat{j}+3\hat{k}$. **(06 Marks)**

b. Find the value of λ so that the vectors $\vec{a} = 2\hat{i}-3\hat{j}+\hat{k}$, $\vec{b} = \hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c} = \hat{j}+\lambda\hat{k}$ are coplanar. **(07 Marks)**

c. Prove that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$ **(07 Marks)**

4 a. A particle moves along the curve $r = 2t^2\hat{i} + (t^2 - 4t)\hat{j} + (3t-5)\hat{k}$. Find the components of velocity and acceleration in the direction of the vector $c = \hat{i} - 3\hat{j} + 2\hat{k}$ at $t = 2$. **(06 Marks)**

b. If $\phi = x^2y^2z^3$ and $\vec{f} = 2x\hat{i} + 3y\hat{j} + 4\hat{x}$ find $\vec{f}.\nabla\phi$ and $\vec{f}\times\nabla\phi$ at (1, 1, 1). **(07 Marks)**

c. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point (1, -2, -1) along the vector $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$. **(07 Marks)**

5 a. If $\vec{a}$ is a vector function and ϕ is a Scalar function then show that $\text{curl}(\phi\vec{a}) = \phi(\text{curl }\vec{a}) + \text{grad}\phi \times \vec{a}$. **(06 Marks)**

b. If $\vec{r} = x\hat{i} + y\hat{j} + 2\hat{x}$ then show that $\nabla r^n = nr^{n-2}\vec{r}$. **(07 Marks)**

c. Find the constants a, b, c so that the vector field $\vec{f} = (x+2y+az)\hat{i} + (bx-3y-z)\hat{j} + (4x+cy+2z)\hat{k}$ is irrotational. **(07 Marks)**

6 a. Prove that $L\{t^n\} = \frac{n!}{s^{n+1}}$, where n is a positive integer. **(05 Marks)**

b. Find: i) $L\{e^{-2t}\cos^2 t\}$ ii) $L\{2^t\cos^3 t\}$ **(10 Marks)**

c. Find: $L\{te^{-3t}\sin 3t\}$. **(05 Marks)**

7 a. If $L\{f(t)\} = F(s)$ show that $L\left\{\int_0^t f(t)dt\right\} = \frac{F(S)}{S}$. (05 Marks)

b. Find:

i) $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$ ii) $L^{-1}\left[\frac{s+3}{s^2+9s+20}\right]$ (10 Marks)

c. Find: $L^{-1}\left[\frac{2s-1}{s^2+2s+17}\right]$ (05 Marks)

8 a. Using the Laplace transform method, solve the initial value problem.

$\frac{d^2x}{dt^2} - \frac{2dx}{dt} + x = e^{2t}$, $x(0) = 0$, $\frac{dx}{dt}(0) = -1$ (10 Marks)

b. Using Laplace transform method solve

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-t}$

$y(0) = 0 = y'(0)$. (10 Marks)

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