

USN

--	--	--	--	--	--	--	--	--	--

MATDIP301

Third Semester B.E. Degree Examination, Feb./Mar. 2022

Advanced Mathematics – I

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions.

- 1 a. Express the complex number $\frac{2-i\sqrt{3}}{\sqrt{3}+i}$ in the form of $x + iy$. (06 Marks)
- b. Express $\sqrt{3} + i$ in the polar form and hence find modulus and amplitude. (07 Marks)
- c. Prove that $\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^4 = \cos 8\theta + i\sin 8\theta$. (07 Marks)
- 2 a. Find the n^{th} derivative of $e^{ax} \sin(bx + c)$. (06 Marks)
- b. If $y = \sin^{-1} x$ then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$. (07 Marks)
- c. Find the n^{th} derivative of $\frac{x + 3}{(x + 2)(2x + 3)}$. (07 Marks)
- 3 a. Find the angle between the curves $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$. (06 Marks)
- b. Find the pedal equation for the curve $r^2 = a^2 \cos 2\theta$. (07 Marks)
- c. Using Maclaurin's series expand $y = \tan x$ upto the term containing x^5 . (07 Marks)
- 4 a. If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. (06 Marks)
- b. If $u = f(xz, y/z)$, prove that $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} - z\frac{\partial u}{\partial z} = 0$. (07 Marks)
- c. If $x + y + z = u$, $y + z = v$ and $z = uvw$ then find the value of $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. (07 Marks)
- 5 a. Obtain the reduction formula for $\int \sin^n x dx$, where n is a positive integer. (06 Marks)
- b. Evaluate $\int_0^{\pi/2} \sin^3 \theta \cos^7 \theta d\theta$. (07 Marks)
- c. Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} x^2 y dy dx$. (07 Marks)
- 6 a. Evaluate $\int_0^1 \int_0^2 \int_0^3 xyz dz dy dx$. (06 Marks)
- b. Define Beta function and prove that $\beta(m, n) = \beta(n, m)$. (07 Marks)
- c. Using beta and gamma functions evaluate $\int_0^{\pi/2} \sin^4 \theta \cos^5 \theta d\theta$. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and/or equations written eg. 42+8 = 50, will be treated as malpractice.

- 7 a. Solve $\frac{dy}{dx} = \frac{y}{x} + \sin \frac{y}{x}$. (06 Marks)
- b. Solve $(1 + y^2)dx = (\tan^{-1} y - x)dy$. (07 Marks)
- c. Solve $(1 + e^{\frac{x}{y}})dx + e^{\frac{x}{y}}(1 - \frac{x}{y})dy = 0$. (07 Marks)
- 8 a. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + 4$. (06 Marks)
- b. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x x^2$. (07 Marks)
- c. Solve $(D^2 + D + 1)y = x^2 + x + 1$. (07 Marks)
