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18MAT41

Fourth Semester B.E. Degree Examination, Feb./Mar. 2022
Complex Analysis, Probability and Statistical Methods

Time: 3 hrs.

Max. Marks: 100

*Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Use of Statistical Tables is permitted.*

Module-1

- 1 a. Derive Cauchy Riemann equations in Cartesian form. (07 Marks)
b. Find the analytic function whose real part is $e^{2x} (x \cos 2y - y \sin 2y)$. (07 Marks)
c. Determine the analytic function $w = u + iv$, if $v = \log(x^2 + y^2) + x - 2y$ (06 Marks)

OR

- 2 a. Derive Cauchy Riemann equations in polar form. (07 Marks)
b. If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z . (07 Marks)
c. Find the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$. (06 Marks)

Module-2

- 3 a. Discuss the transformation $w = e^z$ and show that it transforms the region between the real axis and a line parallel to real axis at $y = \pi$ into upper half of W -plane. (07 Marks)
b. Find the bilinear transformation which maps the points $Z = 1, i, -1$ into the points $W = i, 0, -i$. (07 Marks)
c. Evaluate $\int_C \frac{Z^2 - Z + 1}{Z - 1} dz$, where C is the circle $|Z| = 1/2$. (06 Marks)

OR

- 4 a. Discuss the transformation $W = Z^2$. (07 Marks)
b. Find the bilinear transformation which maps the points $Z = 0, i, \infty$ onto the points $W = 1, -i, -1$ respectively. (07 Marks)
c. Evaluate $\oint_C \frac{\sin \pi Z^2 + \cos \pi Z^2}{(Z - 1)(Z - 2)} dz$ where C is the circle $|Z| = 3$. (06 Marks)

Module-3

- 5 a. The probability density function of a random variable X is as follows:

X	0	1	2	3	4	5	6
P(X)	K	3K	5K	7K	9K	11K	13K

Find K and i) $P(X < 4)$, ii) $P(3 < X \leq 6)$. (07 Marks)

- b. The probability that a pen manufactured by a company will be defective is $\frac{1}{10}$. If 12 such pens are manufactured, find the probability that,
i) Exactly 2 will be defective
ii) None will be defective
iii) At least two will be defective (Use binomial distribution). (07 Marks)

- c. Fit a Poisson distribution to the set of observations

x	0	1	2	3	4
f	122	60	15	2	1

(06 Marks)

OR

- 6 a. If X is a continuous random variable with probability function given by
 $f(x) = kx, 0 \leq x < 2$
 $= 2k, 2 \leq x \leq 4$
 $= -kx + 6k, 4 \leq x < 6,$

Find the value of K and the mean value of X.

(07 Marks)

- b. If the probability of a bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals more than two get a bad reaction. (Use Poisson distribution). (07 Marks)
- c. In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn for i) more than 2150 hours and ii) less than 1950 hours. (06 Marks)

Module-4

- 7 a. Fit a straight line $y = a + bx$ to the following data:

x	0	1	2	3	4
y	1.0	1.8	3.3	4.5	6.3

(07 Marks)

- b. Find the correlation for the data given below

x	1	2	3	4	5	6
y	6	4	3	5	4	2

(07 Marks)

- c. Two regression equations of the variables x and y are $x = 19.13 - 0.87y$ and $y = 11.64 - 0.50x$, find i) mean value of x, mean value of y and ii) Correlation coefficient between x and y. (06 Marks)

OR

- 8 a. Fit a second degree parabola to the following data:

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

(07 Marks)

- b. Ten competitors in a contest are ranked by two judges as follows:

x	1	6	5	10	3	2	4	9	7	8
y	6	4	9	8	1	2	3	10	5	7

Calculate the rank correlation coefficient.

(07 Marks)

- c. Find the regression equations y on x and x on y using the table of values given below.

x	16	24	32	40	48	56
y	0.39	0.75	1.23	1.91	2.77	3.81

(06 Marks)

Module-5

- 9 a. The joint probability distribution of two random variables X and Y is shown below:

	Y	2	3	4
X				
1		0.06	0.15	0.09
2		0.14	0.35	0.21

Find the marginal distributions of X and Y. Also verify that X and Y are stochastically independent. (07 Marks)

- b. A die was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times. On the assumption of random throwing, do the data indicate an unbiased die? (07 Marks)
- c. A certain stimulus administered to each of 12 patients resulted in the following increase in blood. Pressures: 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4. Can it be calculated that stimulus is accompanied by an increase in blood pressure given that for 11 degrees of freedom the value of $t_{0.05}$ is 2.201? (06 Marks)

OR

- 10 a. A joint probability distribution is given by the following table:

	Y	-3	2	4
X				
1		0.1	0.2	0.2
3		0.3	0.1	0.1

Find i) marginal distributions ii) COV (X, Y). (07 Marks)

- b. The mean life time of a sample of 100 bulbs is 1570 hours with a standard deviation of 120 hours. The company claims that the average life of bulbs produced by it is 1600 hrs. Use appropriate test to verify the acceptance ($Z_{0.05} = 1.96$). (07 Marks)
- c. Fit a Poisson distribution to the following data and test for its goodness of fit at level of significance 0.05 ($\chi^2_{0.05} = 7.82$ for 3 degrees of freedom). Given that,

x	0	1	2	3	4
f	419	352	154	56	19

(06 Marks)
