USN

10MAT41

Fourth Semester B.E. Degree Examination, Feb./Mar. 2022
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: *1 Answer any FIVE full questions, selecting atleast TWO questions from each part.*
2. Use of statistical tables is permitted.

PART – A

1 a. Use Picard's method to find y at the point x = 0.25, 0.5, 0.75 given that $\frac{dy}{dx} = \frac{x^2}{1+y^2}$ with y(0) = 1 by taking two approximations. **(06 Marks)**

b. Using modified Euler's methods find y at x = 20.2 and 20.4 given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with y(20) = 5, taking step size h = 0.2. **(07 Marks)**

c. Given $\frac{dy}{dx} = xy + y^2$ and the data y(0) = 1, y(0.1) = 1.1169, y(0.2) = 1.2773, y(0.3) = 1.5049. Find y at x = 0.4 correct to three decimal places using Milne's predictor – corrector method. Apply corrector formula twice. **(07 Marks)**

2 a. Use Picard's method to find y(0.1) and z(0.1) given that $\frac{dy}{dx} = x + z$, $\frac{dz}{dx} = x - y^2$ and y(0) = 2, z(0) = 1. [Carry out two approximations]. **(10 Marks)**

b. Given $\frac{d^2y}{dx^2} - \frac{x.dy}{dx} - y = 0$ with initial conditions y(0) = 1, y′(0) = 0. Find y(0.2) and y′(0.2) using fourth order Runge – Kutta method. **(10 Marks)**

3 a. Define analytic function. Obtain Cauchy's – Riemann equations in Cartesian form. **(06 Marks)**

b. If f(z) is a regular function of z, show that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$. **(07 Marks)**

c. If w = $\phi + i\psi$ represents the complex potential of an electro static field where $\Psi = x^2 - y^2 + \frac{x}{x^2+y^2}$, find f(z) and hence find ϕ. **(07 Marks)**

4 a. Find the bilinear transformation which maps the points z = 0, 1, ∞ into the points w = –5, –1, 3 respectively. Also find the invariant points. **(06 Marks)**

b. Discuss the transformation $w = e^z$. **(07 Marks)**

c. Evaluate $\int_C \frac{\sin\pi^2 z + \cos\pi^2 z}{(z-1)^2(z-2)} dz$ where c is the circle |z| = 3. **(07 Marks)**

PART – B

5 a. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cdot \sin x$ and $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$. **(06 Marks)**

b. State and prove Rodrigues formula for Legendre's polynomials. **(07 Marks)**

c. Express $f(x) = x^3 + 2x^2 - 4x + 5$ in terms of Legendre's polynomials. **(07 Marks)**

6 a. State axioms of probability. If A and B are any two events in a sample space S then prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. **(06 Marks)**

b. Three students A, B, C write an entrance examination. Their chances of passing are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. Find the probability that :

i) Atleast one of them passes
ii) All of them passes
iii) Atleast two of them passes. **(07 Marks)**

c. Three machines A, B, C produces 50%, 30%, 20% of the items in a factory. The percentages of defective output are 3, 4, 5. If an item is selected at random, what is the probability that it is defective? What is the probability that it is from machine A? **(07 Marks)**

7 a. A random variable X has the following probability function for various values of x.

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2 + K$

Find K. Also find $P(x < 6)$, $P(x \geq 6)$, $P(3 < x \leq 6)$. **(06 Marks)**

b. Obtain mean and variance of binomial distribution. **(07 Marks)**

c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be

i) less than 65
ii) more than 75
iii) between 65 and 75. Given A(1) = 0.3413. **(07 Marks)**

8 a. Explain the terms :

i) Null hypothesis
ii) Level of significance
iii) Type I and Type II errors. **(06 Marks)**

b. In 324 throws of six faced die. An odd number turned up 181 times. Is it reasonable to think that the die is an unbiased one? **(07 Marks)**

c. A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure 5, 2, 8, –1, 3, 0, 6, –2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase in the blood pressure? [$t_{0.05}$ for 11 d.f = 2.201]. **(07 Marks)**

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