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HAMILTONIAN LACEABILITY IN CYCLIC PRODUCT AND BRICK PRODUCT OF CYCLES

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ABSTRACT

A connected graph G is said to be Hamiltonian-t-laceable if there exists a Hamiltonian path between every pair of distinct vertices at a distance't' in G and Hamiltonian-t-laceable if there exist at least one such pair, where t is a positive integer. In this paper we explore Hamiltonian-t*- Laceability properties of the cyclic product C(2n, m) and the Brick product C(2n+1, 3, 2) of cycles.*

Keywords: *Hamiltonian-t^*-laceable graph; Cyclic product; Brick product; Laceability number.*
2010 Mathematics Subject Classification: 05C45, 05C99.

1. INTRODUCTION

Let G be a finite, simple connected undirected graph. Let u and v be two vertices in G. The distance between u and v denoted by d(u,v) is the length of a shortest u-v path in G. G is *Hamiltonian-t-laceable* if there exists a Hamiltonian path between every pair of vertices u and v with d(u,v)=t and *Hamiltonian-t*-laceable* if there exists at least one such pair with d(u,v)=t where t is a positive integer such that 1≤ t ≤ diamG. The concept of Hamiltonian laceability of brick products of even cycles was studied by B. Alspach, C.C. Chen and Kevin Mc Avaney in [1]. In [2], Leena Shenoy and R. Murali have discussed the Hamiltonian-t*-laceability of (m,r)-Brick Product of odd cycles C(2n+1,m,r) for m=2 and r=2 and cyclic product for C(2n,m) for m=1,2.

First, we recall the following definitions.

Definition 1.1. Let m and n be positive integers. Let $C_{2n} = a_0\, a_1 a_2\, a_3 a_4\, a_5 ....a_{2n-1}\, a_0$ denote a cycle of order 2n (n>1). Then, the cyclic product of C_{2n} denoted by C(2n, m) is defined as follows.

For m=1, C(2n, 1) is obtained from C_{2n} by adding chords $a_k(a_{2n-k})$, $1 \le k \le (n-1)$ and $a_k(a_{2n})$, for k = n where the computation is performed under modulo 2n.

For m>1, C(2n, m) is obtained by first taking disjoint union of m copies of C_{2n} namely $C_{2n}(1)$, $C_{2n}(2)$, $C_{2n}(3)$……$C_{2n}(m)$ where for each i=1,2,3,…..m $C_{2n}(i)$ = $a_{i1}\, a_{i2} a_{i3}\, a_{i4} a_{i5}\, a_{i6} \ldots\ldots\ldots a_{i(2n-1)}\, a_{i0}$. Further:

Case(i): If m is even, an edge is drawn to join a_{ij} *to* $a_{(i+1)j}$ for both odd or both even $1 \le i \le (m-1)$, $1 \le j \le 2n$ whereas for odd i and even $1 \le j < 2n$ an edge is drawn to join a_{ij} *to* $a_{m(j+1)}$. Finally an edge is drawn to join $a_{i(2n)}$ *to* a_{m1}.

Case(ii): If m is odd an edge is drawn to join a_{ij} *to* $a_{(i+1)j}$ for both odd or both even $1 \le i \le (m-1)$, $1 \le j \le 2n$ whereas for odd i and even $1 \le j < 2n$ an edge is drawn to join a_{ij} *to* $a_{m(j+2)}$. Finally an edge is drawn to join $a_{i(2n)}$ *to* a_{m2}.

The Cyclic products C(8, 4) and C(8, 5) are shown in Fig 1 and Fig 2.

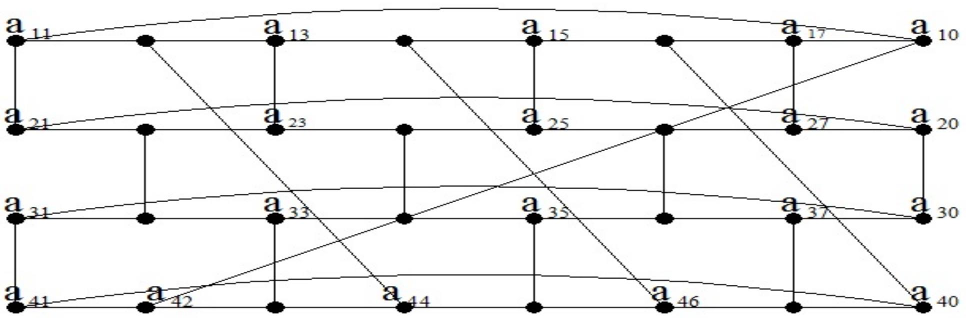

Fig.1 C(8, 4)

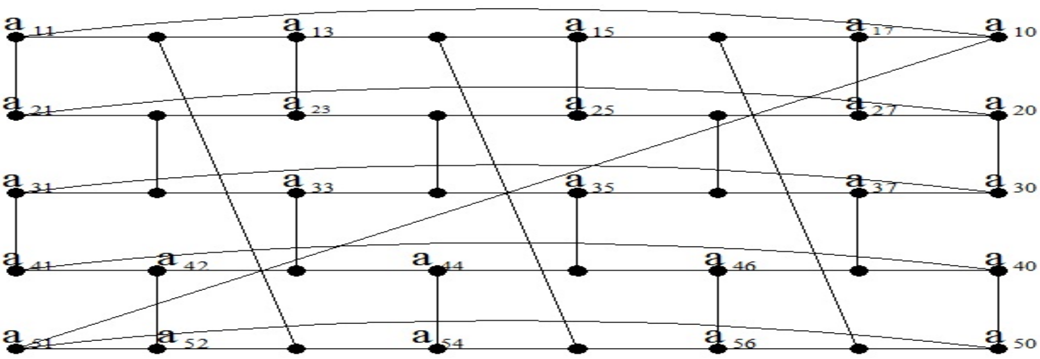

Fig.2 C(8, 5)

Definition 1.2. Let m, n and r be positive integers. Let C_{2n+1}= $a_0\, a_1 a_2\, a_3 a_4\, a_5 \ldots\ldots\ldots a_{2n}\, a_0$ denote a cycle of order 2n+1 (n>1). The (m,r)-brick product of C_{2n+1}, denoted by C(2n+1,m,r) is defined for m=1, we require that 1< r < 2n. Then C(2n+1,m,r) is obtained from C_{2n+1} by adding chords $a_k(a_{k+r})$, $0 \le k \le 2n$ where the computation is performed under modulo 2n+1.

For $m > 1$, C(2n+1, m, r) is obtained by first taking the disjoint union of m copies C_{2n+1} namely C_{2n+1} (1), $C_{2n+1}(2)$, $C_{2n+1}(3)$…….. $C_{2n+1}(m)$ where for each i= 1,2,3……m $C_{2n+1}(i)$ = $a_{i1}\, a_{i2} a_{i3}\, a_{i4} a_{i5}\, a_{i6}.........a_{i(2n)}\, a_{i0}$. Further:

Case(i): If m is odd and 1< r <2n where r is defined as r={(2n+1) j} +2, j≥0, an edge is drawn to join a_{ij} to $a_{(i+1)j}$ for both odd or both even 1≤ i ≤ (m-1), 0 ≤ j ≤ 2n whereas for each odd 1≤ i ≤ (m-1) and even 1≤ j < 2n an edge is drawn to join a_{ij} $to\, a_{m(j+1)}$. Finally an edge is drawn to join $a_{i(2n)}$ $to\, a_{m(2n+r)}$.

Case(ii): If m is even and 1< r <2n where r is defined as r={(2n+1) j} +3, j≥0, an edge is drawn to join a_{ij} to $a_{(i+1)j}$ for both odd or both even 1≤ i ≤ (m-1), 0 ≤ j ≤ 2n whereas for each odd 1≤ i ≤ (m-1) and even 1≤ j < 2n an edge is drawn to join a_{ij} $to\, a_{m(j+2)}$. Finally an edge is drawn to join $a_{i(2n)}$ to $a_{m(2n+r)}$.

The brick product C(11,3,2) is shown in Fig 3.

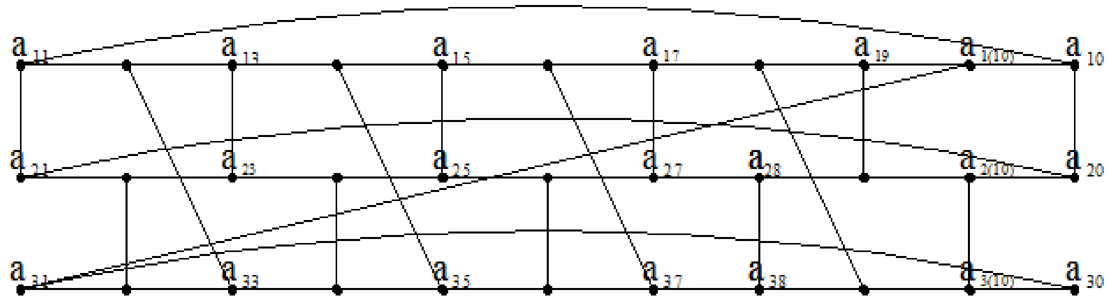

Fig.3 C(11, 3, 2)

Definition 1.3. Let u and v be two distinct vertices in a connected graph G. Then u and v are attainable in G if there exists a Hamiltonian path in G between u and v. We write *(u,v) is attainable* in G.

Definition 1.4. Let a_i and a_j be any two distinct vertices in a connected graph G. Let E' be a minimal set of edges not in G and P be a path in G, such that P ∪ E' is a Hamiltonian path in G from a_i to a_j. Then | E'| is called the t*-laceability number $\lambda^*_{(t)}$ of (a_i, a_j) and the edges in E' are called the t^*- laceability edges with respect to (a_i, a_j).

2. RESULTS

In [2], Leena N.Shenoy and R. Murali proved the following results.

Theorem 2.1. C(2n, 1) is Hamiltonian-t-laceable, 1 ≤ t ≤ diamG.

Theorem 2.2. Let G= C(2n, 2). Then

(i) G is Hamiltonian-t*-laceable for odd t , $1 \leq t \leq (n+1)$ with $\lambda^*_{(t)} = 1$

(ii) G is Hamiltonian-t*-laceable for even t, $1 \leq t \leq (n+1)$ with $\lambda^*_{(t)} = 2$.

We now prove the following results.

Theorem 2.3. Let G=C(2n,m). If n > 3 and even, $m \geq 3$ and $(2n-m) \geq 2$, then

(i) G is Hamiltonian-t*-laceable for t =1.

(ii) G is Hamiltonian-t*-laceable for all $2 \leq t \leq n$ with $\lambda^*_{(t)} = 1$.

(iii) G is Hamiltonian-t*-laceable for t = n+1 with $\lambda^*_{(t)} = 2$.

Proof. Consider G = C (2n, m) with vertices

$a_{11}, a_{12}, a_{13}, a_{14}, a_{15}\ldots\ldots\ldots a_{1(2n-1)}, a_{10}, a_{11}, a_{12},\ldots\ldots\ldots a_{2(2n-1)}, a_{20}, a_{31}, a_{32}\ldots\ldots\ldots$

$\ldots a_{3(2n-1)}, a_{30}, a_{41}, a_{42}\ldots\ldots\ldots a_{4(2n-1)}, a_{40}, a_{51}, a_{52}, a_{53}\ldots\ldots\ldots a_{m(2n-1)}, a_{m0}$

Let $P_{s1} : a_{11} - a_{12} - a_{13} - a_{14} - a_{15}\ldots\ldots\ldots a_{1(2n-1)} - a_{10}$

$P_{s2} : a_{21} - a_{22} - a_{23} - a_{24} - a_{25}\ldots\ldots\ldots a_{2(2n-1)} - a_{20}$

$P_{s3} : a_{31} - a_{32} - a_{33} - a_{34} - a_{35}\ldots\ldots\ldots a_{3(2n-1)} - a_{30}$

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$P_{sm} : a_{m1} - a_{m2} - a_{m3} - a_{m4} - a_{m5}\ldots\ldots\ldots a_{m(2n-1)} - a_{m0}$ be *m* sub paths in G.

Let diamG = n+1and let a_{11} and a_{1i} be the vertices in P_{s1}. We have the following cases.

Case (i). For t=1.

Let i = 2. Then (a_{11}, a_{1i}) is attainable and the path:

$P : a_{11} - a_{21} - a_{22} - a_{23} - a_{24}\ldots\ldots\ldots a_{2(2n)} - a_{3(2n)} - a_{3(2n-1)} - a_{3(2n-2)}\ldots\ldots\ldots a_{31} - a_{41}\ldots\ldots$

$\ldots a_{m1} - a_{m(2n)} - a_{m(2n-1)} - a_{m(2n-2)}\ldots\ldots\ldots a_{m2} - a_{1(2n)} - a_{1(2n-1)} - a_{1(2n-2)}\ldots\ldots\ldots a_{1i}$

is a Hamiltonian path.

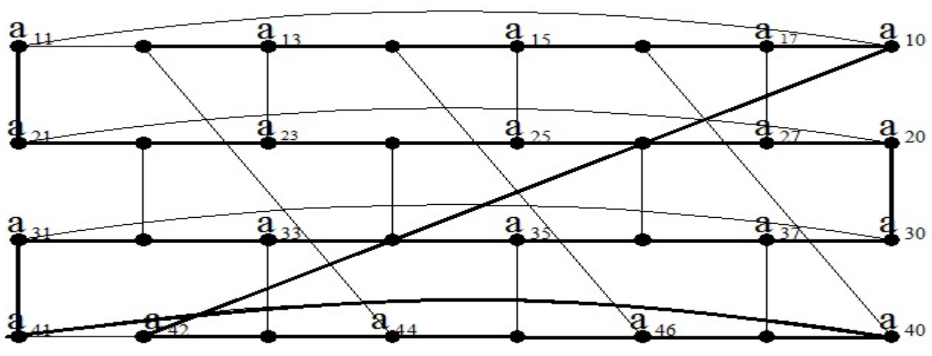

Fig.4 Hamiltonian laceable path from a_{11} to a_{12} in C(8,4)

Case (ii). For $2 \leq t \leq n$.

Let $2 \leq i \leq (n+1)$. Then (a_{11}, a_{1i}) is attainable for each and the path:

$P: a_{11} - a_{21} - a_{22} - a_{23} - a_{24} \ldots\ldots\ldots a_{2(2n)} - a_{3(2n)} - a_{3(2n-1)} - a_{3(2n-2)} \ldots\ldots\ldots a_{31} - a_{41} \ldots\ldots\ldots a_{m1}$
$- a_{m(2n)} - a_{m(2n-1)} - a_{m(2n-2)} \ldots\ldots\ldots a_{m2} - a_{1(2n)} - a_{1(2n-1)} - a_{1(2n-2)} \ldots\ldots\ldots a_{1(i+1)} - a_{12} - a_{13} - a_{14} \ldots\ldots a_{1i}$

is a Hamiltonian path with t* laceability edge $(a_{1(i+1)}, a_{12})$.

Case (iii). For t=n+1.

Let i = n+2 for $n \geq 4$, consider a vertex a_{mi} on P_{sm} Then (a_{11}, a_{mi}) is attainable and the path

$P: a_{11} - a_{21} - a_{22} - a_{23} - a_{24} \ldots\ldots\ldots a_{2(2n)} - a_{3(2n)} - a_{3(2n-1)} - a_{3(2n-2)} \ldots\ldots\ldots a_{31} - a_{41} - a_{42} - a_{43} \ldots\ldots\ldots a_{4(2n)}$
$- a_{5(2n)} - a_{5(2n-1)} - a_{5(2n-2)} \ldots\ldots\ldots a_{m1} - a_{m2} - a_{m3} - a_{m4} - a_{12} - a_{13} - a_{14} - a_{15} \ldots\ldots\ldots a_{1(2n)} - a_{m(2n)} - a_{m(2n-1)}$
$- a_{m(2n-2)} \ldots\ldots\ldots a_{m(1+i)} - a_{m5} - a_{m6} \ldots\ldots\ldots a_{mi}$

is a Hamiltonian path with t* laceability edge $(a_{1(2n)}, a_{m(2n)})$ *and* $(a_{m(i+1)}, a_{m5})$.

Hence the proof. □

For n=3, we have the following result.

Theorem 2.4. Consider G=C(2n,m). If n = 3 and even, $m \geq 3$ and $(2n-m) \geq 2$ then G is Hamiltonian-t*-laceable for t = 4 with $\lambda^*_{(t)} = 1$.

Proof. Let G=C(6,m). If i = n+2 for n = 3, consider a vertex a_{m5} on P_{sm}. Then (a_{11}, a_{mi}) is attainable and the path:

$P: a_{11} - a_{12} - a_{13} - a_{14} - a_{15} \ldots\ldots\ldots a_{20} - a_{30} - a_{35} - a_{34} - a_{33} - a_{31} - a_{41} - a_{42} - a_{43} \ldots\ldots\ldots a_{46}$
$- a_{56} - a_{55} - a_{54} \ldots\ldots\ldots a_{m1} - a_{m2} - a_{m3} - a_{m4} - a_{12} - a_{13} - a_{14} - a_{15} \ldots\ldots\ldots a_{10} - a_{m5} - a_{m0}$

is a Hamiltonian path with t* laceability edge (a_{10}, a_{m5}).

Hence the proof. □

Theorem 2.5. Let G=C(2n,m). If $n \geq 3$ and odd, $m \geq 3$ and $(2n-m) \geq 3$ then

(i) G is Hamiltonian-t*-laceable for t =1.

(ii) G is Hamiltonian-t*-laceable for all $2 \leq t \leq n$ with $\lambda^*_{(t)} = 1$.

(iii) G is Hamiltonian-t*-laceable for t = n+1 with $\lambda^*_{(t)} = 1$.

Proof. Consider G=C(2n, m) with vertices:

$a_{11}, a_{12}, a_{13}, a_{14}, a_{15} \ldots\ldots\ldots a_{1(2n-1)}, a_{10}, a_{11}, a_{12}, \ldots\ldots\ldots a_{2(2n-1)}, a_{20}, a_{31}, a_{32} \ldots\ldots\ldots$
$\ldots a_{3(2n-1)}, a_{30}, a_{41}, a_{42} \ldots\ldots\ldots a_{4(2n-1)}, a_{40}, a_{51}, a_{52}, a_{53} \ldots\ldots\ldots a_{m(2n-1)}, a_{m0}$

$Let \quad P_{s1}: a_{11} - a_{12} - a_{13} - a_{14} - a_{15} \ldots\ldots\ldots\ldots a_{1(2n-1)} - a_{10}$

$P_{s2}: a_{21} - a_{22} - a_{23} - a_{24} - a_{25} \ldots\ldots\ldots\ldots a_{2(2n-1)} - a_{20}$

$P_{s3}: a_{31} - a_{32} - a_{33} - a_{34} - a_{35} \ldots\ldots\ldots\ldots a_{3(2n-1)} - a_{30}$

$------------$

$------------$

$P_{sm}: a_{m1} - a_{m2} - a_{m3} - a_{m4} - a_{m5} \ldots\ldots\ldots\ldots a_{m(2n-1)} - a_{m0}$ be *m* sub paths in G.

Let diam G = n+1 and a_{11} and a_{1i} be the vertices in P_{s1}. We have the following cases.

Case (i). For t=1.

Let i = 2. Then (a_{11}, a_{12}) is attainable and the path:

$P: a_{11} - a_{21} - a_{22} - a_{23} - a_{24} \ldots\ldots\ldots\ldots a_{2(2n)} - a_{3(2n)} - a_{3(2n-1)} - a_{3(2n-2)} - a_{3(2n-3)} - a_{3(2n-4)} \ldots\ldots\ldots\ldots\ldots$

$\ldots\ldots a_{31} - a_{41} - a_{42} \ldots\ldots\ldots\ldots\ldots\ldots a_{4(2n)} - a_{5(2n)} - a_{5(2n-1)} - a_{5(2n-2)} \ldots\ldots\ldots\ldots a_{m(2n)} - a_{m(2n-1)} - a_{m(2n-2)} \ldots\ldots\ldots$

$\ldots\ldots a_{m1} - a_{1(2n)} - a_{1(2n-1)} - a_{1(2n-2)} \ldots\ldots\ldots\ldots\ldots a_{1i}$

is a Hamiltonian path.

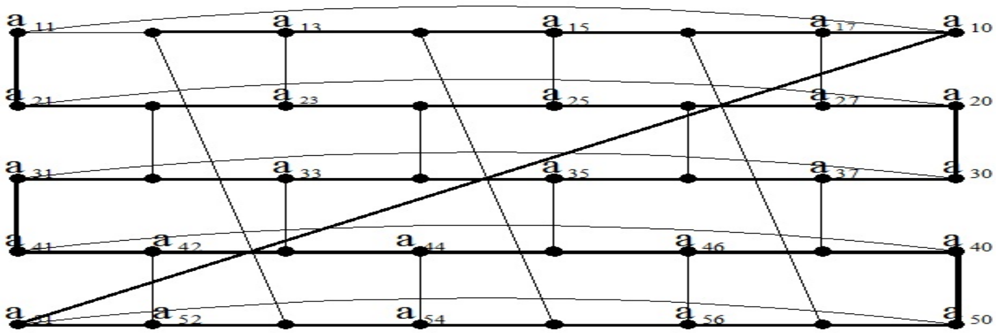

Fig.5 Hamiltonian Path from a_{11} to a_{12} in C(8,5)

Case (ii). For 2 ≤ t ≤ n.

Let 2 ≤ i ≤ (n+1). Then (a_{11}, a_{1i}) is attainable for each *i* and the path:

$P: a_{11} - a_{21} - a_{22} - a_{23} - a_{24} \ldots\ldots\ldots\ldots a_{2(2n)} - a_{3(2n)} - a_{3(2n-1)} - a_{3(2n-2)} - a_{3(2n-3)} - a_{3(2n-4)} \ldots\ldots\ldots\ldots\ldots$

$\ldots\ldots a_{31} - a_{41} - a_{42} \ldots\ldots\ldots\ldots\ldots a_{4(2n)} - a_{5(2n)} - a_{5(2n-1)} - a_{5(2n-2)} \ldots\ldots\ldots\ldots a_{m1} - a_{1(2n)} - a_{1(2n-1)} - a_{1(2n-2)} \ldots\ldots$

$\ldots\ldots\ldots\ldots\ldots a_{1(i+1)} - a_{12} - a_{13} - a_{14} \ldots\ldots\ldots\ldots\ldots\ldots a_{1i}$

is a Hamiltonian path with t* laceability edge $(a_{1(i+1)}, a_{12})$.

Case (iii). For t=n+1

Sub Case (i). Let i = n+2 for even n ≥ 4, consider a vertex a_{mi} on P_{sm}. Then (a_{11}, a_{mi}) is attainable and the path:

$P: a_{11} - a_{12} - a_{13} - a_{14} - a_{15} \ldots\ldots\ldots a_{1(i-2)} - a_{m(i-1)} - a_{m(i-2)} - a_{m(i-3)} \ldots\ldots\ldots a_{m1} - a_{1(2n)} - a_{1(2n-1)}$
$- a_{1(2n-2)} - a_{1(i-1)} - a_{2(i-1)} - a_{2(i-2)} - a_{2(i-3)} \ldots\ldots\ldots a_{21} - a_{2(2n)} - a_{2(2n-1)} - a_{2(2n-2)} - a_{2(2n-3)} \ldots\ldots\ldots$
$\ldots a_{2i} - a_{3i} - a_{3(i-1)} - a_{3(i-2)} \ldots\ldots\ldots a_{3(2n)} - a_{3(2n-1)} - a_{3(2n-2)} \ldots\ldots\ldots a_{3(i+1)} - a_{4(i+1)} - a_{4i} - a_{4(i-1)} \ldots\ldots\ldots$
$\ldots\ldots a_{41} - a_{4(2n)} \ldots\ldots\ldots a_{(m-1)2n} - a_{(m-1)(2n-1)} - a_{(m-1)(2n-2)} \ldots\ldots\ldots a_{(m-1)2n} - a_{(m-1)(2n-1)} - a_{(m-1)(2n-2)} \ldots\ldots$
$\ldots\ldots a_{(m-1)(m+i-3)} - a_{m(2n)} - a_{m(2n-1)} \ldots\ldots\ldots a_{mi}$

is a Hamiltonian path with t* laceability edge $(a_{(m-1)(m+i-3)}, a_{m(2n)})$.

Sub Case (ii). Let i = n+2 for odd n ≥ 3. Consider a vertex a_{mi} on P_{sm}. Then (a_{11}, a_{mi}) is attainable and the path:

$P: a_{11} - a_{12} - a_{13} - a_{14} - a_{15} \ldots\ldots\ldots a_{1(2n)} - a_{m1} - a_{m2} - a_{m3} \ldots\ldots\ldots a_{m(i-1)} - a_{(m-1)(i-1)} - a_{(m-1)(i-2)}$
$- a_{(m-1)(i-3)} - a_{(m-1)(i-4)} \ldots\ldots\ldots a_{(m-1)1} - a_{(m-2)1} - a_{(m-2)2} - a_{(m-2)3} \ldots\ldots\ldots a_{(m-2)(2n)} - a_{(m-3)(2n)}$
$- a_{(m-3)(2n-1)} - a_{(m-3)(2n-2)} - a_{(m-3)(2n-3)} \ldots\ldots\ldots a_{21} - a_{(m-1)i} - a_{(m-1)(i+1)} - a_{(m-1)(i+2)} \ldots\ldots\ldots a_{(m-1)(2n)}$
$- a_{m(2n)} - a_{m(2n-1)} - a_{m(2n-2)} \ldots\ldots\ldots a_{mi}$

is a Hamiltonian path with t* laceability edge $(a_{21}, a_{(m-1)i})$.

Hence the Proof. □

Theorem 2.6. Let G = C(2n+1, 3, 2) for n ≥ 3, then

(i) G is Hamiltonian-t*- laceable for t=1

(ii) G is Hamiltonian-t*- laceable for 2 ≤ t ≤ (n+1) with $\lambda^*_{(t)}=1$

Proof. Consider G = C(2n+1, 3, 2) with vertices: $a_{11}, a_{12}, a_{13}, a_{14}, a_{15} \ldots\ldots\ldots a_{1(2n-1)} a_{1(2n)} a_{10}, a_{21}, a_{22}, a_{23} \ldots\ldots\ldots a_{2(2n-1)}, a_{2(2n)}, a_{20}, a_{31}, a_{32}, a_{33} \ldots\ldots$
$\ldots\ldots a_{3(2n-1)}, a_{3(2n)}, a_{30}$
under modulo 2n+1.

Let $P_{s1}: a_{11} - a_{12} - a_{13} - a_{14} - a_{15} \ldots\ldots\ldots a_{1(2n-1)} - a_{1(2n)} - a_{10}$

$P_{s2}: a_{21} - a_{22} - a_{23} - a_{24} - a_{25} \ldots\ldots\ldots a_{2(2n-1)} - a_{2(2n)} - a_{20}$

$P_{s3}: a_{31} - a_{32} - a_{33} - a_{34} - a_{35} \ldots\ldots\ldots a_{3(2n-1)} - a_{3(2n)} - a_{30}$ be three sub paths in G.

Let diam G = n+1and a_{11} and a_{1i} be the vertices in P_{s1}.We have the following cases.

Case (i). For t=1.

Let i = 2. Then (a_{11}, a_{12}) is attainable and the path:

$P: a_{11} - a_{10} - a_{1(10)} - a_{19} - a_{18} \ldots\ldots\ldots a_{13} - a_{23} - a_{22} - a_{21} - a_{20} - a_{2(10)} - a_{29} - a_{28} \ldots\ldots\ldots$
$\ldots\ldots a_{24} - a_{34} - a_{35} - a_{36} \ldots\ldots\ldots a_{30} - a_{31} - a_{32} - a_{33} - a_{12}$

is a Hamiltonian path.

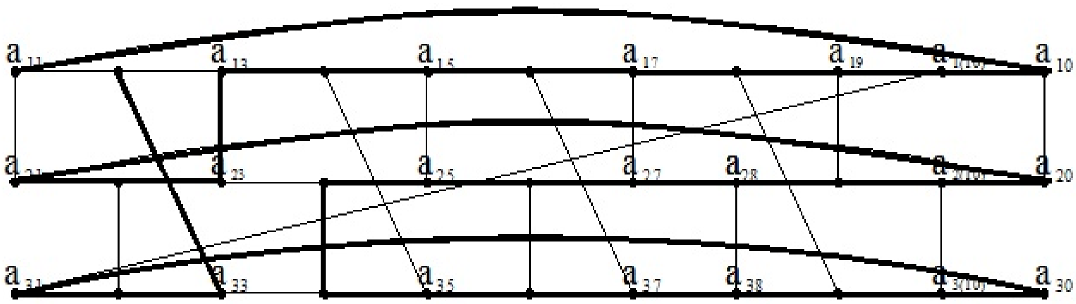

Fig.6 Hamiltonian Path from a_{11} to a_{12} in C(11, 3, 2)

Case (ii). For $2 \le t \le n$

Sub Case (i). For each odd i, $2 < i \le (n+1)$, (a_{11}, a_{1i}) is attainable and the path

$$P: a_{11} - a_{10} - a_{1(2n)} - a_{19} - a_{18} \ldots\ldots\ldots a_{1(i+1)} - a_{1(i+2)} - a_{2(i+2)} - a_{2(i+1)} - a_{2i} - a_{2(i-1)} \ldots\ldots\ldots$$
$$\ldots a_{21} - a_{20} - a_{2(2n)} - a_{29} - a_{28} \ldots\ldots\ldots a_{2(i+3)} - a_{3(i+3)} - a_{3(i+4)} - a_{3(i+5)} \ldots\ldots\ldots a_{30} - a_{31} - a_{32}$$
$$- a_{33} \ldots\ldots\ldots a_{3(i+2)} - a_{1(i+1)} - a_{12} - a_{13} - a_{14} \ldots\ldots\ldots a_{1i}$$

is a Hamiltonian path with t* laceability edge $(a_{1(i+1)}, a_{12})$.

Sub Case (ii). For each even i between $2 < i \le (n+1)$, (a_{11}, a_{1i}) is attainable and the path:

$$P: a_{11} - a_{10} - a_{1(2n)} - a_{19} - a_{18} \ldots\ldots\ldots a_{1(i+1)} - a_{2(i+1)} - a_{2i} - a_{2(i-1)} \ldots\ldots\ldots a_{21} - a_{20} - a_{2(2n)} - a_{29}$$
$$- a_{28} \ldots\ldots\ldots a_{2(i+2)} - a_{3(i+2)} - a_{3(i+3)} - a_{3(i+4)} \ldots\ldots\ldots a_{30} - a_{31} - a_{32} - a_{33} \ldots\ldots\ldots a_{3(i+1)} - a_{12}$$
$$- a_{13} - a_{14} \ldots\ldots\ldots a_{1i}$$

is a Hamiltonian path with t* laceability edge $(a_{3(i+1)}, a_{12})$.

Case (iii). For t= n+1

Sub Case (i). Let i = n+2 for odd $n \ge 3$, consider a vertex a_{3i} on P_{s3}. Then (a_{11}, a_{3i}) is attainable and the path:

$$P: a_{11} - a_{12} - a_{13} - a_{14} - a_{15} \ldots\ldots\ldots a_{10} - a_{20} - a_{21} - a_{22} - a_{23} - a_{24} \ldots\ldots\ldots a_{2(i-1)} - a_{3(i-1)} - a_{3(i-2)}$$
$$- a_{3(i-3)} - a_{3(i-4)} \ldots\ldots\ldots a_{31} - a_{30} - a_{3(2n)} - a_{2(2n)} - a_{2(2n-1)} - a_{2(2n-2)} \ldots\ldots\ldots a_{2i} - a_{3(2n-1)} - a_{3(2n-2)}$$
$$- a_{3(2n-3)} \ldots\ldots\ldots a_{3i}$$

is a Hamiltonian path with t* laceability edge $(a_{2i}, a_{3(2n-1)})$.

Sub Case (ii). Let i = n+2 for even $n \ge 4$, consider a vertex a_{3i} on P_{s3}.

Then (a_{11}, a_{3i}) is attainable and the path:

$$P: a_{11} - a_{12} - a_{13} - a_{14} - a_{15} \ldots\ldots\ldots a_{10} - a_{20} - a_{21} - a_{22} - a_{23} - a_{24} \ldots\ldots\ldots$$
$$\ldots a_{2(2n)} - a_{3(2n)} - a_{30} - a_{31} - a_{32} \ldots\ldots\ldots a_{3(i-1)} - a_{3(2n-1)} - a_{3(2n-2)}$$
$$- a_{2(2n-3)} \ldots\ldots\ldots a_{3i}$$

is a Hamiltonian path with t* laceability edge $(a_{3(i-1)}, a_{3(2n-1)})$.

Hence the proof. □

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