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ON FUZZY WEAKLY COMPLETELY PRIME Γ - IDEALS OF TERNARY Γ - SEMI GROUPS

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ABSTRACT

In this paper we introduce the concept of ternary Γ - semi group and we proved some properties of prime Γ -ideals and fuzzy weakly completely prime Γ -ideals of ternary Γ - semi groups. It is proved that, in a ternary Γ - semi group S if $A \subseteq S$, then the following statements are equivalent (1) A is prime Γ -ideal of a ternary Γ - semi group S (2) The characteristic function C_A of A is a fuzzy weakly completely prime Γ -ideal of S.

Key words: *Γ - semi group, Ternary Γ - semi group, prime Γ -ideals, fuzzy weakly completely prime Γ -ideals.*

1. INTRODUCTION AND PRELIMINARIES

Lehmer.D.H [4] gave the definition of a ternary semi group. Banach showed that a ternary semi group doesn't necessarily reduce to an ordinary semi group. J.Los [2] showed that any ternary semi group however may be embedded in an ordinary semi group in such a way that the operation in ternary semi groups is an (ternary) extension of the (binary) operation of the containing semi group. Kim.J [3], Lyapin.E.S. [5] and F.M.Sioson [1] have also studied the properties of ternary semi groups. M.K.Sen [6] defined the concepts of Γ -semi group. It is known that Γ -semi group is a generalization of semi group. Many classical notions of semi groups have been extended to Γ -semi groups. In this paper we introduced the concept of ternary Γ - semi group and discussed the results on prime Γ -ideals and fuzzy weakly completely prime Γ -ideals of ternary Γ - semi groups.

Definition: 1.1.1 Let S and Γ be two non-empty sets. If there exists a mapping $S \times \Gamma \times S \to S$, defined by $(a,\alpha,b) = a\alpha b$. Then S is called Γ - semi group when S satisfies the identities $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a,b,c \in S$ and $\alpha,\beta \in \Gamma$.

Definition: 1.1.2 A ternary Γ - semi group is an algebraic structure $(S,\Gamma,*)$ such that S is a non-empty set and $*: S\times\Gamma\times S\times\Gamma\times S \to S$ is a ternary operation satisfying the following associative law $(a\alpha b\beta c)\gamma d\delta e = a\alpha(b\beta c\gamma d)\delta e = a\alpha b\beta(c\gamma d\delta e)$ for all $a,b,c,d,e \in S$, $\alpha,\beta,\gamma,\delta \in \Gamma$.

Definition: 1.1.3 A non-empty subset A of a ternary Γ - semi group S is called a ternary sub Γ - semi group of S if $A\Gamma A\Gamma A \subseteq A$.

Definition: 1.1.4 A non – empty subset A of a ternary Γ - semi group S is called a left (right, lateral) Γ -ideal of S if $S\Gamma S\Gamma A \subseteq A$ $(A\Gamma S\Gamma S \subseteq A,\ S\Gamma A\Gamma S \subseteq A)$

Definition: 1.1.5 A non–empty subset A of S is called a two sided Γ- ideal of S if it is both left Γ- ideal and right Γ- ideal of S.

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Definition: 1.1.6 A fuzzy set μ of a ternary Γ - semi group S is a fuzzy Γ -ideal of S if it is fuzzy left Γ- ideal, fuzzy right Γ- ideal and fuzzy lateral Γ- ideal of S .

Definition: 1.1.7 Let A be a subset of a ternary Γ - semi group S . Then the characteristic function of A is defined by

$$C_A(x)=\begin{cases}1 & if \quad x\in A\\ 0 & if \quad x\notin A\end{cases}$$

Definition: 1.1.8. A subset A of a ternary Γ - semi group S is said to be a prime ideal of S if $x\Gamma y\Gamma z\subseteq A$ implies $x\in A$ or $y\in A$ or $z\in A$.

Definition: 1.1.9 A fuzzy Γ -ideal μ of a ternary Γ - semi group S is called a fuzzy weakly completely prime Γ -ideal of S if $\mu(x)\geq\mu(x\alpha y\beta z)$ or $\mu(y)\geq\mu(x\alpha y\beta z)$ or $\mu(z)\geq\mu(x\alpha y\beta z)$ for all $x,y,z\in S$ and $\alpha,\beta\in\Gamma$

Definition: 1.1.10 A fuzzy Γ -ideal μ of a ternary Γ - semi group S is called a fuzzy prime Γ -ideal of S if $\inf\limits_{\alpha,\beta\in\Gamma}\mu(x\alpha y\beta z)\geq\max\{\mu(x),\mu(y),\mu(z)\}$ for all $x,y,z\in S$ and $\alpha,\beta\in\Gamma$.

Based on these preliminaries we prove some results on prime Γ -ideals and fuzzy weakly completely prime Γ -ideals of ternary Γ - semi groups.

1.2. MAIN RESULTS

Theorem: 1.2.1 Let μ be a non-empty fuzzy subset of a ternary Γ - semi group S . Then $1-\mu$ is a fuzzy ternary sub- Γ - semi group of S if and only if μ is a fuzzy weakly completely prime Γ -ideal of S .

Proof: Let $1-\mu$ be a fuzzy ternary sub Γ - semi group of S . Let $x,y,z\in S$ and $\alpha,\beta\in\Gamma$. Then

$$\begin{aligned}1-\mu(x\alpha y\beta z)&=1-\min\{\mu(x),\mu(y),\mu(z)\}\\ &\geq\min\{1-\mu(x),1-\mu(y),1-\mu(z)\}\end{aligned}$$

$$1-\mu(x\alpha y\beta z)\geq 1-\max\{\mu(x),\mu(y),\mu(z)\}$$

$$-\mu(x\alpha y\beta z)\geq-\max\{\mu(x),\mu(y),\mu(z)\}$$

$$\mu(x\alpha y\beta z)\leq\max\{\mu(x),\mu(y),\mu(z)\}$$

ie. $\max\{\mu(x),\mu(y),\mu(z)\}\geq\mu(x\alpha y\beta z)$

$\therefore\ \mu(x)\geq\mu(x\alpha y\beta z)$ or $\mu(y)\geq\mu(x\alpha y\beta z)$ or $\mu(z)\geq\mu(x\alpha y\beta z)$

Hence μ is a fuzzy weakly completely prisime Γ -ideal of S .

Conversely, assume that μ is a fuzzy weakly completely prime Γ -ideal of S . Then we have $\mu(x)\geq\mu(x\alpha y\beta z)$ or $\mu(y)\geq\mu(x\alpha y\beta z)$ or $\mu(z)\geq\mu(x\alpha y\beta z)$.

Consider

$$\max\{\mu(x),\mu(y),\mu(z)\}\geq\mu(x\alpha y\beta z)$$

$$1-\max\{\mu(x),\mu(y),\mu(z)\}\leq 1-\mu(x\alpha y\beta z)$$

$$\min\{1-\mu(x),1-\mu(y),1-\mu(z)\}\leq 1-\mu(x\alpha y\beta z)$$

$$\mu^{/}(x\alpha y\beta z \geq \min\{\mu^{/}(x), \mu^{/}(y), \mu^{/}(z)\}$$

$\therefore\ \mu^{/} = 1-\mu$ is a fuzzy ternary sub-Γ- semi group of S.

Theorem: 1.2.2 Let $\{\mu_i : i \in I\}$ be a family of fuzzy weakly completely prime Γ-ideal of a ternary Γ- semi group S. Then $\bigcap\limits_{i\in I}\mu_i$ is a fuzzy weakly completely prime Γ-ideal of S.

Proof: Let $\{\mu_i : i \in I\}$ be a family of fuzzy weakly completely prime Γ-ideal of a ternary Γ- semi group S. Then we have $\mu_i(x) \geq \mu_i(x\alpha y\beta z)$ or $\mu_i(y) \geq \mu_i(x\alpha y\beta z)$ or $\mu_i(z) \geq \mu_i(x\alpha y\beta z)$ for $x, y, z \in S$ and $\alpha, \beta \in \Gamma, i \in I$, Then

$$\bigcap_{i\in I}\mu_i(x\alpha y\beta z) = \inf\{\mu_i(x\alpha y\beta z) : i \in I\}$$

$$\therefore\ \bigcap_{i\in I}\mu_i(x\alpha y\beta z) \leq \inf\{\mu_i(x) : i \in I\}$$

$$\text{Or } \bigcap_{i\in I}\mu_i(x\alpha y\beta z) \leq \inf\{\mu_i(y) : i \in I\}$$

$$\text{Or } \bigcap_{i\in I}\mu_i(x\alpha y\beta z) \leq \inf\{\mu_i(z) : i \in I\}$$

Hence $\bigcap\limits_{i\in I}\mu_i$ is fuzzy weakly ternary completely prime Γ-ideal of S.

Theorem: 1.2.3 Let S be a ternary Γ-semi group and μ be a non-empty fuzzy subset of S. Then the following are equivalent:

(1) μ is a fuzzy weakly completely prime Γ-ideal of S

(2) For any $t \in [0,1]$, μ_t (if it is non-empty) is a prime Γ-ideal of S

Proof: Let μ be a fuzzy weakly completely prime Γ-ideal of a ternary Γ- semi group S. Then we have $\mu(x) \geq \mu(x\alpha y\beta z)$ or $\mu(y) \geq \mu(x\alpha y\beta z)$ or $\mu(z) \geq \mu(x\alpha y\beta z)$ for all $x, y, z \in S$ and $\alpha, \beta \in \Gamma$. Let $t \in [0,1]$ be such that μ_t is non-empty. Let $x, y, z \in S$, $x\Gamma y\Gamma z \subseteq \mu_i$. Then $\mu(x\alpha y\beta z) \geq t$ for all $\alpha, \beta \in \Gamma$. Since μ is a fuzzy weakly completely prime Γ-ideal of a ternary Γ- semi group S, so, we have $\mu(x) \geq \mu(x\alpha y\beta z)$ or $\mu(y) \geq \mu(x\alpha y\beta z)$ or $\mu(z) \geq \mu(x\alpha y\beta z)$ for all $x, y, z \in S$ and $\alpha, \beta \in \Gamma$. Then $\mu(x) \geq t$ or $\mu(y) \geq t$ or $\mu(z) \geq t$ which implies that $x \in \mu_t$ or $y \in \mu_t$ or $z \in \mu_t$. Hence μ_t is a prime Γ-ideal of S.

Conversely, let us suppose that μ_t is a prime Γ-ideal of a ternary Γ- semi group S. Let $\mu(x\alpha y\beta z) = t$. Then $\mu(x\alpha y\beta z) \geq t$ for all $\alpha, \beta \in \Gamma$. Hence μ_t is non-empty and $x\Gamma y\Gamma z \subseteq \mu_t$. Since μ_t is a prime Γ-ideal of S,

we have $x \in \mu_t$ or $y \in \mu_t$ or $z \in \mu_t$. Then $\mu(x) \geq t$ or $\mu(y) \geq t$ or $\mu(z) \geq t$ which implies that $\mu(x) \geq \mu(x\alpha y\beta z)$ or $\mu(y) \geq \mu(x\alpha y\beta z)$ or $\mu(z) \geq \mu(x\alpha y\beta z)$. Hence μ is a fuzzy weakly completely prime Γ-ideal of a ternary Γ- semi group S.

Theorem: 1.2.4 Let A be a non-empty subset of a ternary Γ- semi group S and C_A be the characteristic function of A. Then A is a left Γ-ideal of S if and only if C_A is a fuzzy left Γ-ideal of S.

Proof: Assume that A is a left Γ-ideal of S. Let $x, y, z \in S$. If $z \in A$, then
$C_A(x) = C_A(y) = C_A(z) = 1$ and since $x\alpha y\beta z \in S\Gamma S\Gamma A \subseteq A$

$\therefore$ We have $C_A(x\alpha y\beta z) = 1 = C_A(x) \wedge C_A(y) \wedge C_A(z)$

If $x \notin A$ or $y \notin A$ or $z \notin A$, then $C_A(x) = 0$ or $C_A(y) = 0$ or $C_A(z) = 0$

And we have $C_A(x\alpha y\beta z) = 0 = C_A(x) \wedge C_A(y) \wedge C_A(z)$
$\therefore C_A(x\alpha y\beta z) \geq C_A(x) \wedge C_A(y) \wedge C_A(z)$
$\therefore C_A$ is a ternary sub Γ - semi group of S.

And let $x, y, z \in S$. If $z \in A$ then $C_A(x) = C_A(y) = C_A(z) = 1$ and since $x\alpha y\beta z \in S\Gamma S\Gamma A \subseteq A$, we have $C_A(x\alpha y\beta z) = 1 = C_A(z)$. If $x \notin A$ or $y \notin A$ or $z \notin A$, then $C_A(x) = 0$ or $C_A(y) = 0$ or $C_A(z) = 0$ and we have $C_A(x\alpha y\beta z) \geq 0 = C_A(z) \Rightarrow C_A(x\alpha y\beta z) \geq C_A(z)$
$\therefore C_A$ is a fuzzy left Γ-ideal of S.

Theorem: 1.2.5 Let A be a non-empty subset of a ternary Γ - semi group S and C_A be the characteristic function of A. Then A is a right Γ-ideal (lateral Γ-ideal, Γ-ideal) of S if and only if C_A is a fuzzy right Γ-ideal (fuzzy lateral Γ-ideal, fuzzy Γ-ideal) of S.

Proof: Similar to the proof of Theorem 1.2.4.

Theorem: 1.2.6 Let S be a ternary Γ - semi group and A be a non-empty subset of S. Then the following are equivalent:

(1) A is prime Γ-ideal of a ternary Γ - semi group S

(2) The characteristic function C_A of A is a fuzzy weakly completely prime Γ-ideal of S.

Proof: Let A be a prime Γ-ideal of a ternary Γ - semi group S and C_A be the characteristic function of A. Since $A \neq \phi$, so, C_A is non-empty. Let $x, y, z \in S$. Suppose $x\Gamma y\Gamma z \subseteq A$. Then $C_A(x\alpha y\beta z) = 1$ for $\alpha, \beta \in \Gamma$. Since A is a prime Γ-ideal of S, $x \in A$ or $y \in A$ or $z \in A$ which implies that $\mathrm{C}_A(x) = 1$ or $\mathrm{C}_A(y) = 1$ or $\mathrm{C}_A(z) = 1$. Hence $\mathrm{C}_A(x) \geq \mathrm{C}_A(x\alpha y\beta z)$ or $C_A(y) \geq C_A(x\alpha y\beta z)$ or $C_A(z) \geq C_A(x\alpha y\beta z)$. Suppose $x\Gamma y\Gamma z \not\subset A$. Then $C_A(x\alpha y\beta z) = 0$ for $\alpha, \beta \in \Gamma$. Since A be a prime Γ-ideal of S, $x \notin A$ or $y \notin A$ or $z \notin A$ which implies that $C_A(x) = 0$ or $C_A(y) = 0$ or $C_A(z) = 0$. Hence $C_A(x) \geq C_A(x\alpha y\beta z)$ or $C_A(y) \geq C_A(x\alpha y\beta z)$ or $C_A(z) \geq C_A(x\alpha y\beta z)$. Consequently C_A is a fuzzy weakly completely prime Γ-ideal of S.

Conversely, let the characteristic function C_A of A is a fuzzy weakly completely prime Γ-ideal of S. Then C_A is a fuzzy Γ-ideal of S. By the theorem 2.1.4, A is an Γ-ideal of S. Let $x, y, z \in S$ be such that $x\Gamma y\Gamma z \subseteq A$. Then $C_A(x\alpha y\beta z) = 1$. Let if possible $x \notin S$ and $y \notin S$ and $z \notin S$. Then $C_A(x) = C_A(y) = C_A(z) = 0$ which implies $C_A(x) < C_A(x\alpha y\beta z)$ and $C_A(y) < C_A(x\alpha y\beta z)$ and $C_A(z) < C_A(x\alpha y\beta z)$. This contradicts our assumption that C_A is a fuzzy weakly completely prime Γ-ideal of S.

Hence A is prime Γ-ideal of S.

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