CBCS SCHEME

USN | | | | | | | | | | | **18CS36**

Third Semester B.E. Degree Examination, Feb./Mar. 2022
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Prove that for any propositions p, q, r the compound proposition $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a Tautology. **(08 Marks)**

b. Prove the logical equivalence without using truth table:
$p \to (q \to r) \Leftrightarrow (p \wedge q) \to r$ **(05 Marks)**

c. Find whether the following argument is valid. No engineering student of first or second semester studies logic.
Anil is an Engineering student who studies logic
$\therefore$ Anil is not in second semester **(07 Marks)**

OR

2 a. Give a direct proof and an indirect proof for the given statement. "If 'n' is an odd integer, then n + 9 is an even integer". **(06 Marks)**

b. Prove the given logical equivalence problem using laws of logic.
$(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg (q \vee p)$. **(07 Marks)**

c. Verify the given argument is valid or not?
$p \to (q \to r)$
$p \vee \neg s$
q
$\therefore s \to r$ **(07 Marks)**

Module-2

3 a. Prove that for each $n \in z^+$
$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = 1/6\, n(n+1)(2n+1)$ **(07 Marks)**

b. Find the number of permutation of the letter of the word "MASSASAUGA". In how many of there all four 'A's are together? How many of them begin with 'S'? **(06 Marks)**

c. Find how many distinct four digit integers one can make from the digit 1, 3, 3, 7, 7, 8. **(07 Marks)**

OR

4 a. Determine the co-efficient of xyz^2 in the expansion of $(2x - y - z)^4$. **(06 Marks)**

b. In how many ways can 10 identical pencils be distributed among 5 children in following cases:
i) There are no restrictions.
ii) Each child gets atleast one pencil.
iii) The youngest child gets at least two pencils. **(07 Marks)**

c. Find the number of arrangements of all the letters in "TALLAHASSEE"? How many of these arrangement have no adjacent 'A's? **(07 Marks)**

Module-3

5 a. Let $f: R \to R$ be defined by

$$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \leq 0 \end{cases}$$

find $f^{1}(0)$, $f^{1}(1)$, $f^{1}(3)$, $f^{1}(-3)$, $f^{1}(-6)$, $f^{1}([-5, 5])$. **(07 Marks)**

b. On the set Z^{-1} a relation 'R' is defined by aRb if and only if "a divides b (exactly)" verify that 'R' is equivalence relation. **(06 Marks)**

c. Draw the Hasse diagram representing the positive divisor of 36. **(07 Marks)**

OR

6 a. Let A = {1, 2, 3, 4, 5} define relation 'R' on A×A by (X_1Y_1) R (X_2Y_2) if and only if $X_1 + Y_1 = X_2 + Y_2$.

i) Verify 'R' is a equivalence relation on A×A

ii) Determine the partition of A×A induced by R. **(07 Marks)**

b. Let A = {1, 2, 3, 4, 6} and 'R' be a relation on 'A' defined by aRb if and only if "a is multiple of b" represent the relation 'R' as a matrix, draw its diagraph and relation R. **(06 Marks)**

c. Let f, g, h be a function from R to R defined by f(x) = x + 2, g(x) = x – 2, h(x) = 3x for $\forall x \in R$ find gof, fog, fof, gog, foh, fohog. **(07 Marks)**

Module-4

7 a. How many integers between 1 and 300 (inclusive) are

i) Divisible by atleast one of 5, 6, 8

ii) Divisible by none of 5, 6, 8. **(07 Marks)**

b. Find the rook polynomial for the 3 × 3 board by using the expansion formula. **(07 Marks)**

c. Solve the recurrence relation $a_n - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$ given that $a_0 = 2$. **(06 Marks)**

OR

8 a. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(06 Marks)**

b. Solve the recurrence relation $a_n = 2(a_{n-1} - a_{n-2})$ for $n \geq 2$ given that $a_0 = 1$ and $a_1 = 2$. **(07 Marks)**

c. Compute derangement of d_4, d_5, d_6, d_7. **(07 Marks)**

Module-5

9 a. Define Isomorphism. Verify the given two graphs are Isomorphic (Fig.Q.9(a)). **(07 Marks)**

Fig.Q.9(a)

b. "A tree with 'n' vertices has n – 1 edges". Prove this. Define a tree. **(06 Marks)**

c. Construct an optimal prefix code for the given set of frequencies, 20, 28, 4, 17, 12, 7. **(07 Marks)**

OR

10 a. Explain complete graph, Bipartite graph, subgraph, regular graph, spanning subgraph, minimally connected graph, with example for each. **(07 Marks)**

b. Apply merge sort to the given list -1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3. **(06 Marks)**

c. Obtain an optimal prefix code for the message "LETTER RECEIVED" indicate the code. **(07 Marks)**

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