MATDIP401

# Fourth Semester B.E. Degree Examination, July/August 2021

## Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

***Note: Answer any FIVE full questions.***



1 a. Find the angle between any two diagonals of a cube. **(06 Marks)**

b. Find the equation of the plane which passes through the points (0, 1, 1), (1, 1, 2) and (–1, 2 –2). **(07 Marks)**

c. Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ are coplanar and find their common point. **(07 Marks)**

2 a. Find the angle between the planes $x+y+2z-3=0$ and $2x+3y+3z-4=0$. **(06 Marks)**

b. Find the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ **(07 Marks)**

c. Find the image of the point (1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$. **(07 Marks)**

3 a. If $\vec{A}=2\hat{i}-3\hat{j}-\hat{k}$ and $\vec{B}=\hat{i}+4\hat{j}-2\hat{k}$, find the angle between the vectors $\vec{A}$ and $\vec{B}$. **(06 Marks)**

b. If $\vec{a}=\hat{i}+\hat{j}-\hat{k}$, $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$, evaluate (i) $[\vec{a}\ \vec{b}\ \vec{c}]$ (ii) $\vec{a}\times(\vec{b}\times\vec{c})$ **(07 Marks)**

c. Find the constant $\lambda$ such that the vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}-3\hat{k}$ and $3\hat{i}+\lambda\hat{j}+5\hat{k}$ are coplanar. **(07 Marks)**

4 a. A particle moves on the curve $x=2t^2$, $y=t^2-4t$, $z=3t-5$, where t is the time. Find the components of velocity and acceleration at $t=1$ in the direction of $\hat{i}-3\hat{j}+2\hat{k}$. **(06 Marks)**

b. If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ and $r=|\vec{r}|$, show that $\nabla r^n=nr^{n-2}\,\vec{r}$. **(07 Marks)**

c. Find a unit vector normal to the surface $x^3+y^3+3xyz=3$ at (1, 2, –1). **(07 Marks)**

5 a. If $\vec{A}=\text{grad}(x^3+y^3+z^3-3xyz)$, find $\text{div}\,\vec{A}$ and $\text{curl}\,\vec{A}$. **(06 Marks)**

b. Find the constant a, b, c so that $\vec{F}=(x+2y+az)\hat{i}+(bx-3y-z)\hat{j}+(4x+cy+2z)\hat{k}$ is irrotational. **(07 Marks)**

c. Find angle between the surfaces $x^2+y^2+z^2=9$ and $x=z^2+y^2-3$ at (2, –1, 2). **(07 Marks)**

6 Find the Laplace transform of:

a. $e^{-3t}(2\cos 5t-3\sin 5t)$ **(05 Marks)**

b. $\sin 3t\sin 2t+t\cos t$ **(05 Marks)**

c. $\frac{\cos at-\cos bt}{t}$ **(05 Marks)**

d. $e^{2t}+4t^3-2\sin 3t+3\cos 3t+2^{-t}$ **(05 Marks)**

7 Find the inverse Laplace transform of

a. $\dfrac{s^2-3s+4}{s^3}$ **(05 Marks)**

b. $\dfrac{2}{(s-1)(s-2)(s-3)}$ **(05 Marks)**

c. $\log\left[\dfrac{s^2+1}{s(s+1)}\right]$ **(05 Marks)**

d. $\dfrac{2s-3}{s^2+4s+13}$ **(05 Marks)**

8 a. Solve using Laplace transformation method $y''+2y'-3y=\sin t$, $y(0)=y'(0)=0$. **(10 Marks)**

b. By Laplace transform method solve the equation $\dfrac{d^2y}{dt^2}+4\dfrac{dy}{dt}+3y=e^{-t}$ with $y(0)=1$, $y'(0)=1$. **(10 Marks)**

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