# CBCS SCHEME

USN | | | | | | | | | | |

15MATDIP41

## Fourth Semester B.E. Degree Examination, July/August 2021
## Additional Mathematics – II

Time: 3 hrs. Max. Marks: 80

***Note: Answer any FIVE full questions.***



1 a. Determine the rank of the matrix $A=\begin{bmatrix}0 & 1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 3 & 1 & 0 & 2\\ 1 & 1 & -2 & 0\end{bmatrix}$ by applying elementary row transformations. **(05 Marks)**

b. Find the inverse of the matrix $\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$ using Cayley Hamilton theorem. **(05 Marks)**

c. Solve by Gauss elimination method

$2x+y+4z=12$

$4x+11y-z=33$

$8x-3y+2z=20$ **(06 Marks)**

2 a. Find the eigen values of $A=\begin{bmatrix}7 & -2 & 0\\ -2 & 6 & -2\\ 0 & -2 & 5\end{bmatrix}$ **(05 Marks)**

b. Solve the system of equations by Gauss elimination method.

$x+y+z=9$

$x-2y+3z=8$

$2x+y-z=3$ **(06 Marks)**

c. Find the rank of the matrix by reducing it to echelon form.

$$\begin{bmatrix}-2 & -1 & -3 & -1\\ 1 & 2 & 3 & 1\\ 1 & 0 & 1 & 1\\ 0 & 1 & 1 & 1\end{bmatrix}$$ **(05 Marks)**

3 a. Solve $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+5y=0$ subject to $\frac{dy}{dx}=2$, $y=1$ at $x=0$. **(05 Marks)**

b. Solve $(4D^4-4D^3-23D^2+12D+36)y=0$. **(05 Marks)**

c. Solve by the method of variation of parameters $\frac{d^2y}{dx^2}+y=\tan x$. **(06 Marks)**

4 a. Solve $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=e^{2x}+\cos 2x$. **(05 Marks)**

b. Solve $y''+2y'+y=2x+x^2$ **(05 Marks)**

c. Using the method of undetermined coefficients, solve $y''-5y'+6y=e^{3x}+x$ **(06 Marks)**

5 a. Find the Laplace transform of (i) $\dfrac{e^{-at} - e^{-bt}}{t}$ (ii) $\sin 5t \cos 2t$ **(05 Marks)**

b. Find the Laplace transform of

$$f(t) = \begin{cases} E, & 0 < t < \frac{a}{2} \\ -E, & \frac{a}{2} < t < a \end{cases} \quad \text{where } f(t + a) = f(t)$$ **(06 Marks)**

c. Express $f(t) = \begin{cases} t, & 0 < t < 4 \\ 5, & t > 4 \end{cases}$ in terms of unit step function and hence find $L[f(t)]$. **(05 Marks)**

6 a. Express $f(t) = \begin{cases} \cos t, & 0 < t < \pi \\ \cos 2t, & \pi < t < 2\pi \\ \cos 3t, & t > 2\pi \end{cases}$ in terms of unit step function and hence find its Laplace transform. **(06 Marks)**

b. Find the Laplace Transform of (i) $t \sin at$ (ii) $t^5 e^{4t}$ **(05 Marks)**

c. If $f(t) = t^2, 0 < t < 2$ and $f(t + 2) = f(t)$ for $t > 2$, find $L[f(t)]$. **(05 Marks)**

7 a. Find the inverse Laplace Transform of $\dfrac{2s - 1}{s^2 + 4s + 29}$. **(05 Marks)**

b. Find the inverse Laplace transform of $\cot^{-1}\left(\dfrac{s}{a}\right)$. **(05 Marks)**

c. Solve by using Laplace Transforms $\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} + 4y = e^{-t}$; $y(0) = 0$, $y'(0) = 0$. **(06 Marks)**

8 a. Solve the initial value problem $y'' + 4y' + 3y = e^{-t}$ conditions with $y(0) = 1$, $y'(0) = 1$ using Laplace Transforms. **(06 Marks)**

b. Find the inverse Laplace Transform of $\dfrac{s + 2}{s^2(s + 3)}$ **(05 Marks)**

c. Find the inverse Laplace Transform of $\log\left[\dfrac{s^2 + 4}{s(s + 4)(s - 4)}\right]$ **(05 Marks)**

9 a. A box contains 3 white, 5 black and 6 red balls. If a ball is drawn at random, what is the probability that it is either red or white? **(05 Marks)**

b. The probability that a person A solves the problem is 1/3, that of B is 1/2 and that of C is 3/5. If the problem is simultaneously assigned to all of them what is the probability that the problem is solved? **(05 Marks)**

c. Three machines A, B and C produce respectively 60%, 30%, 10% of the total number of items of a factory. The percentages of defective output of these machines are respectively 2%, 3% and 4%. An item is selected at random and is found defective. Find the probability that the item was produced by machine C. **(06 Marks)**

10 a. State and prove Baye's theorem. **(05 Marks)**

b. If A and B are events with $P(A \cup B) = \dfrac{3}{4}$, $P(\overline{A}) = \dfrac{2}{3}$ and $P(A \cap B) = \dfrac{1}{4}$, find $P(A)$, $P(B)$ and $P(A \cap \overline{B})$. **(05 Marks)**

c. Three students A, B, C, write an entrance examination. Their chances of passing are 1/2, 1/3 and 1/4 respectively. Find the probability that
(i) atleast one of them passes (ii) all of them pass (iii) atleast two of them passes. **(06 Marks)**

* * * * *