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CBCS SCHEME

USN

18MATDIP31

Third Semester B.E. Degree Examination, July/August 2021
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions.

1 a. Show that $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$. **(07 Marks)**

b. Express $1-i\sqrt{3}$ in polar form and hence find its modulus and amplitude. **(06 Marks)**

c. Express $\frac{1}{1-\cos\theta+i\sin\theta}$ in the form a + ib and also find its conjugate. **(07 Marks)**

2 a. Define dot product between two vectors A and B. Find the sine of the angle between the vectors $\vec{A}=2\hat{i}-2\hat{j}+\hat{k}$ and $\vec{B}=\hat{i}-2\hat{j}+2\hat{k}$. **(07 Marks)**

b. If $\vec{A}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{B}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{C}=3\hat{i}+\hat{j}$, find the value of p such that $\vec{A}-p\vec{B}$ is perpendicular to $\vec{C}$. **(06 Marks)**

c. Find $\vec{a}\cdot(\vec{b}\times\vec{c})$, $\vec{b}\times(\vec{a}\times\vec{c})$ and $\vec{c}\cdot(\vec{a}\times\vec{b})$ where $\vec{a}=\hat{i}+\hat{j}-\hat{k}$, $\vec{b}=2\hat{i}-\hat{j}+2\hat{k}$, $\vec{c}=3\hat{i}-\hat{j}-\hat{k}$. **(07 Marks)**

3 a. Obtain the Maclaurin's series expansion of $\log(\sec x)$ upto the terms containing x^6. **(07 Marks)**

b. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then using Euler's theorem, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$. **(06 Marks)**

c. If $u=f(x-y, y-z, z-x)$, prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$. **(07 Marks)**

4 a. Obtain the Maclaurin's series expansion of the function $\sqrt{1+\sin 2x}$ upto x^4. **(07 Marks)**

b. If $u=e^{\frac{x^2y^2}{x+y}}$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3u\log u$ using Euler's theorem. **(06 Marks)**

c. If $u=\frac{yz}{x}$, $v=\frac{zx}{y}$, $w=\frac{xy}{z}$ then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)}=4$ **(07 Marks)**

5 a. A particle moves along a curve $x=3t^2$, $y=t^3-4t$, $z=3t+4$ where t is the time variable. Determine the components of velocity and acceleration vectors at t = 2 in the direction $\hat{i}-2\hat{j}+2\hat{k}$. **(07 Marks)**

b. Find the unit normal vector to the surface $xy^3z^2=4$ at the point (–1, –1, 2). **(06 Marks)**

c. Show that the vector field $\vec{F}=(2x+yz)\hat{i}+(4y+zx)\hat{j}-(6z-xy)\hat{k}$ is irrotational. Also find ϕ such that $\vec{F}=\nabla\phi$. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

6 a. Find $\text{div}\,\vec{F}$ and $\text{Curl}\,\vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. (07 Marks)

b. If $\vec{F} = (3x^2y - z)\hat{i} + (xz^3 + y^4)\hat{j} - 2x^3z^2\hat{k}$ then find $\nabla\cdot\vec{F}$, $\nabla\times\vec{F}$ and $\nabla\cdot(\nabla\times\vec{F})$ at $(2, -1, 0)$. (06 Marks)

c. Determine the constant 'a' such that the vector $(2x+3y)\hat{i} + (ay - 3z)\hat{j} + (6x - 12z)\hat{k}$ is Solenoidal. (07 Marks)

7 a. Obtain a reduction formula for $\int_0^{\pi/2} \cos^n x\,dx (n > 0)$. (07 Marks)

b. Evaluate $\int_0^a x^4\sqrt{a^2 - x^2}\,dx$. (06 Marks)

c. Evaluate $\int_1^5 \int_1^{x^2} x(x^2 + y^2)\,dxdy$. (07 Marks)

8 a. Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x\,dx\ (n > 0)$. (07 Marks)

b. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$ (06 Marks)

c. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dydxdz$ (07 Marks)

9 a. Solve $(2x^3 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy = 0$ (07 Marks)

b. Solve $\dfrac{dy}{dx} - y\tan x = y^2 \sec x$. (06 Marks)

c. Solve $3x(x + y^2)dy + (x^3 - 3xy - 2y^3)dx = 0$ (07 Marks)

10 a. Solve $\dfrac{dy}{dx} + y\cot x = \sin x$. (07 Marks)

b. Solve $(x + 3y - 4)dx + (3x + 9y - 2)dy = 0$ (06 Marks)

c. Solve $[1 + (x + y)\tan y]\dfrac{dy}{dx} + 1 = 0$ (07 Marks)

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