CBCS SCHEME

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18MAT41

Fourth Semester B.E. Degree Examination, July/August 2021
Complex Analysis, Probability and Statistical Methods

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions.***

1 a. Derive the Cauchy-Riemann equations in Cartesian form. **(07 Marks)**

b. Find the analytic function whose real part is $u = x^3 - 3xy^2 + 3x + 1$, also find its imaginary part. **(07 Marks)**

c. Show that the function $u = \sin x \cosh y + 2\cos x \sinh y + x^2 - y^2 + 4xy$ is harmonic. **(06 Marks)**

2 a. Find the analytic function whose imaginary part is $e^{-x}(x\cos y + y\sin y)$. **(06 Marks)**

b. Find the analytic function $f(z) = u + iv$. Given that $u + v = x^3 - y^3 + 3x^2y - 3xy^2$. **(07 Marks)**

c. If $f(z) = u + iv$ is an analytic function, prove that $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right]|f(z)|^2 = 4|f^1(z)|^2$. **(07 Marks)**

3 a. Discuss the conformal transformation $W = z^2$. **(07 Marks)**

b. Find a bilinear transformation that maps the points (1, i, -1) in z-plane into (i, 0, -i) in W-plane respectively. **(07 Marks)**

c. Evaluate $\oint_c \frac{e^{2z}}{(z+1)(z-2)} dz$, where C is the circle $|z| = 3$. **(06 Marks)**

4 a. Find a bilinear transformation which maps the points 0, 1, ∞ in Z-plane in to -5, -1, 3 in W- plane. **(07 Marks)**

b. State and prove the Cauchy's integral formula. **(06 Marks)**

c. Evaluate $\oint_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where C is the circle $|z| = 3$. **(07 Marks)**

5 a. The pdf of a random variable X is given by the following table

x	;	0	1	2	3	4	5	6
P(x)	;	k	3k	5k	7k	9k	11k	13k

Find k, Also find $p(x \geq 5)$ and $p(3 < x \leq 6)$ **(06 Marks)**

b. If 20% bolts produced by a machine are defective. Calculate the probability that out of 7 randomly selected bolts not more than 1 is defective, atmost two are defective? **(07 Marks)**

c. In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for
   i) 10 minutes or more
   ii) less than 10 minutes
   iii) between 10 to 12 minutes. **(07 Marks)**

6 a. A random variable x has the density function $P(x)=\begin{cases} Kx^2, & 0\le x\le 3 \\ 0, & \text{otherwise} \end{cases}$. Find K, also find (i) $P(x \le 1)$ (ii) $P(1 \le x \le 2$ iii) $P(x \le 2)$ iv) $P(x > 1)$ v) $P(x > 2)$. **(07 Marks)**

b. In 800 families with 5 children each how many family would be expected to have (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys (iv) atmost 2 girls by assuming probability for boy and girls to be equal. **(07 Marks)**

c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and SD 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 (iii) between 65 and 75. Given $\phi(1) = 0.3413$. **(06 Marks)**

7 a. Fit a straight line of the form $y = ax + b$ to the following data :

x	;	0	5	10	15	20	25
y	;	12	15	17	22	24	30

(06 Marks)

b. Compute the coefficient of correlation and equations of regression lines from the data :

x	;	1	2	3	4	5	6	7
y	;	9	8	10	12	11	13	14

(07 Marks)

c. Find the rank correlation coefficient for the data :

x	;	80	72	45	55	56	58	69	65	76	85
y	;	84	70	56	50	48	60	64	65	82	81

(07 Marks)

8 a. Show that if θ is the angle between the lines of regression, then $\tan\theta = \dfrac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\dfrac{1-r^2}{r}\right)$. **(06 Marks)**

b. Fit a parabola of the form $y = ax^2 + bx + c$ to the following data :

x	;	0	1	2	3	4	5
y	;	1	3	7	13	21	31

(07 Marks)

c. Fit a curve of the form $y = ax^b$ to the following data :

x	;	3	4	5	6	7
y	;	6	9	10	11	12

(07 Marks)

9 a. The joint probability distribution of two random variables X and Y is as follows :

X \ Y	-4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Compute E(X), E(Y), E(XY), $E(X^2)$, $E(Y^2)$, σ_x, σ_y, cov(X,Y) and ρ(X, Y). **(07 Marks)**

b. Define the terms :
(i) Null hypothesis
(ii) Type – I and Type – II errors
(iii) Levels of significance. **(06 Marks)**

c. Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches ($t_{0.05} = 2.262$ for 9 d.f) **(07 Marks)**

10 a. If X and Y are independent random variables X takes the values 2, 5, 7, with probability $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$ and Y takes the values 3, 4, 5 with probability $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$

i) Find the marginal probability and joint probability distribution of X and Y

ii) Show that Cov (X, Y) = 0. **(07 Marks)**

b. A coin is tossed 1000 times and head turns up 540 times. Decide on the hypothesis that the coin is unbiased. **(06 Marks)**

c. Five dice were thrown 96 times and the numbers 1, 2 or 3 appearing on the face of the dice follows the frequency distribution as below :

No. of dice showing 1, 2 or 3	5	4	3	2	1	0
Frequency	7	19	35	24	8	3

Test the hypothesis that the data follows a binomial distribution ($\chi^2 = 11.07$ for 5 d.f) **(07 Marks)**

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