CBCS SCHEME

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15MAT41

Fourth Semester B.E. Degree Examination, July/August 2021
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:80

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Find y at x = 0.4 correct to 4 decimal places given $\frac{dy}{dx} = 2xy + 1$; y(0) = 0 applying Taylor's series method upto third degree term. **(05 Marks)**

b. Using modified Euler's method find y(0.2) correct to four decimal places solving the equation $y' = x - y^2$, y(0) = 1 taking h = 0.1. Use modified Euler's formula twice. **(05 Marks)**

c. Use fourth order Runge – Kutta method to solve $(x+y)\frac{dy}{dx} = 1$, y(0.4) = 1 at x = 0.5 correct to four decimal places. **(06 Marks)**

2 a. Using Runge-Kutta method of fourth order, find y(0.2) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, y(0) = 1 by taking h = 0.2. **(05 Marks)**

b. Apply Milne's method to find y at x = 1.4 correct to four decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and the following data y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649, y(1.3) = 2.7514. **(05 Marks)**

c. Find the value of y at x = 4.4 by applying Adams – Bashforth method given that $5x\frac{dy}{dx} + y^2 - 2 = 0$ with the initial values of y : $y_0 = 1$, $y_1 = 1.0049$, $y_2 = 1.0097$, $y_3 = 1.0142$ corresponding to the values of x : $x_0 = 4$, $x_1 = 4.1$m, $x_2 = 4.2$, $x_3 = 4.3$. **(06 Marks)**

3 a. Apply Milne's predictor – corrector method to compute y(0.4) given the differential equation $y'' + 3xy' - 6y = 0$ and the following table of initial values. **(05 Marks)**

x	0	0.1	0.2	0.3
y	1	1.03995	1.13803	1.29865
y′	0.1	0.6955	1.258	1.873

b. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cdot \sin x$. **(05 Marks)**

c. Express $f(x) = 4x^3 + 6x^2 + 7x + 2$ in terms of Legendre polynomials. **(06 Marks)**

4 a. Given $y'' - xy' - y = 0$ with the initial conditions y(0) = 1, y′(0) = 0, compute y(0.2) using fourth order Runge – Kutta method. **(05 Marks)**

b. Prove the Rodrigues formula $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$. **(05 Marks)**

c. Obtain the series solution of Bessel's differential equation $x^2y'' + xy' + (x^2 + n^2)y = 0$. **(06 Marks)**

5 a. State and prove Cauchy's – Riemann equation in polar form. **(05 Marks)**

b. Discuss the transformation $W = Z^2$. **(05 Marks)**

c. Using Cauchy's residue theorem evaluate :

$$\int_C \frac{z\cos z}{(z - \frac{\pi}{2})^3} dz \quad \text{where} \quad C : |z-1| = 1.$$ **(06 Marks)**

6 a. Find an analytical function whose real part is $e^{-x}\left[(x^2-y^2)\cos y+2xy\sin y\right]$. (05 Marks)

b. Evaluate : $\int_C \frac{e^{2z}}{(z+1)(z-2)}\,dz$ where C is the circle $|z|=3$. (05 Marks)

c. Find the bilinear transformation which maps the points $Z = 1, i, -1$ into $w = 0, 1, \infty$. (06 Marks)

7 a. A random variate X has the following probability function for various values of x

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

Find : i) K ii) Evaluate $P(x<6)$ $P(x\geq 6)$ and $P(0<x<5)$. (05 Marks)

b. Find the mean and standard deviation of the exponential distribution. (05 Marks)

c. The joint probability distribution table for two random variables X and Y as follows :

X \ Y	−2	−1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine :
i) Marginal distribution of X and Y
ii) Expectation of X
iii) S.D of Y
iv) Covariance of X and Y
v) Correlation of X and Y. (06 Marks)

8 a. A random variable x has the following density function :

$$f(x)=\begin{cases}Kx^2, & 0<x<3\\ 0, & \text{otherwise}\end{cases}$$

Evaluate : i) K ii) $P(1<x<2)$ iii) $P(x\leq 1)$ iv) $P(x>1)$ v) Mean. (05 Marks)

b. In a quiz contest of answering 'Yes' or 'No' what is the probability of guessing atleast 6 answers correctly out of 10 questions asked? Also find the probability of the same if there are 4 options for a correct answer. (05 Marks)

c. In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and S.D of the distribution. It is given that if :

$$P(Z)=\frac{1}{\sqrt{2\pi}}\int_0^z e^{-z^2/2}\,dz$$

then $A(-0.4958)=0.19$ and $A(1.405)=0.42$. (06 Marks)

9 a. The weights of 1500 ball bearings are normally distributed with a mean of 635gms and S.D of 1.36gms. If 300 random samples of size 36 are drawn from this population, determine the expected mean and S.D of the sampling distribution of means if sampling is done : i) with replacement ii) without replacement. **(05 Marks)**

b. Two athletes A and B were tested according to the time (in seconds) to run a particular race with the following results.

Athlete A	28	30	32	33	33	29	34
Athlete B	29	30	30	24	27	29	

Test whether you can discriminate between the two Athletes. ($t_{0.05} = 2.2$ and $t_{0.02} = 2.72$ for 11d.f). **(05 Marks)**

c. A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand if he does not study one night, he is 60% sure not to study the next night. In the long run how often does he study? **(06 Marks)**

10 a. The mean and S.D of the maximum loads supported by 60 cables are 11.09 tonnes and 0.73 tonnes respectively. Find : i) 95% ii) 99% confidence limits for mean of the maximum loads of all cables produced by the company. **(05 Marks)**

b. Fit a Poisson distribution for the following data and test the goodness of fit given that $\chi^2_{0.05} = 7.815$ for 3d.f.

x	0	1	2	3	4
f	122	60	15	2	1

(05 Marks)

c. Show that $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ is a regular stochastic matrix. Also find the associated unique fixed probability vector. **(06 Marks)**

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