

CBCS SCHEME



USN

--	--	--	--	--	--	--	--	--	--

17MAT31

Third Semester B.E. Degree Examination, July/August 2021 Engineering Mathematics - III

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions.

- 1 a. Obtain the Fourier series for the function

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Hence deduce $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

(08 Marks)

- b. Find the Fourier series for the function $f(x) = 2x - x^2$ in $0 < x < 3$. (06 Marks)
 c. Obtain the constant term and the first sine and cosine terms of the Fourier for y using the following table :

x :	0	1	2	3	4	5
y :	4	8	15	7	6	2

(06 Marks)

- 2 a. Obtain the Fourier series for the function $f(x) = |\cos x|$, $-\pi < x < \pi$. (08 Marks)
 b. Find the Half range cosine series for $f(x) = x(\ell - x)$, $0 \leq x \leq \ell$. (06 Marks)
 c. Express y as a Fourier series upto first harmonic given :

x :	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y :	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

(06 Marks)

- 3 a. If $f(x) = \begin{cases} 1 - x^2, & |x| < 0 \\ 0, & |x| \geq 1 \end{cases}$

Find the Fourier transform of $f(x)$ and hence find the value of $\int_0^{\infty} \left(\frac{x \cos x - \sin x}{x^3} \right) dx$

(08 Marks)

- b. Find the Fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx$ ($m > 0$)

(06 Marks)

- c. Find $Z_T^{-1} \left[\frac{3z^2 + 2z}{(5z-1)(5z+2)} \right]$.

(06 Marks)

- 4 a. Find the Fourier transform of

$$f(x) = \begin{cases} 1 - |x|, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases} \text{ and hence evaluate } \int_0^{\infty} \frac{\sin^2 t}{t^2} dt.$$

(08 Marks)

- b. Find the Z - transform of $2n + \sin \left(\frac{n\pi}{4} \right) + 1$.

(06 Marks)

- c. Solve by using Z - transforms $Y_{n+2} - 4 Y_n = 0$ given that $Y_0 = 0, Y_1 = 2$.

(06 Marks)

- 5 a. Obtain the lines of regression and hence find the coefficient of correlation for the data :

x :	1	3	4	2	5	8	9	10	13	15
y :	8	6	10	8	12	16	16	10	32	32

(08 Marks)

- b. Fit a Second degree parabola in the least Square sense for the following data:

x :	1	2	3	4	5
y :	10	12	13	16	19

(06 Marks)

- c. Use the Regula-Falsi method to obtain the real root of the equation $\cos x = 3x - 1$ correct to 3 decimal places in $(0, 1)$. (06 Marks)

- 6 a. Given the equation of the regression lines $x = 19.13 - 0.87y$, $y = 11.64 - 0.5x$. Compute the mean of x 's , mean of y 's and the coefficient of correlation. (08 Marks)

- b. Fit a curve of the form, $y = a e^{bx}$ for the data:

x :	0	2	4
y :	8.12	10	31.82

(06 Marks)

- c. Using Newton-Raphson method to find a real root of $x \log_{10} x = 1.2$ upto 5 decimal places near $x = 2.5$. (06 Marks)

- 7 a. Given $\sin 45^\circ = 0.7071$, $\sin 50^\circ = 0.7660$, $\sin 55^\circ = 0.8192$, $\sin 60^\circ = 0.8660$, find $\sin 57^\circ$ using an Backward Interpolation formula. (08 Marks)

- b. Applying Lagrange's Interpolation formula inversely find x when $y = 6$ given the data

x :	20	30	40
y :	2	4.4	7.9

(06 Marks)

- c. Using Simpson's $\frac{1}{3}$ rule with Seven ordinates to evaluate $\int_2^8 \frac{dx}{\log_{10} x}$. (06 Marks)

- 8 a. Fit an Interpolating polynomial for the data $u_{10} = 355$, $u_0 = -5$, $u_8 = -21$, $u_1 = -14$, $u_4 = -125$ by using Newton's Divided difference formula and hence find u_2 . (08 Marks)

- b. Use Lagrange's Interpolation formula to fit a polynomial for the data :

x :	0	1	3	4
y :	-12	0	6	12

Hence estimate y at $x = 2$.

(06 Marks)

- c. Evaluate $\int_4^{5.2} \log_e x \, dx$ taking six equal strips by applying Weddle's rule. (06 Marks)

- 9 a. Using Green's theorem, evaluate $\int_C [(y - \sin x)dx + \cos x \, dy]$, where C is the plane triangle

enclosed by the lines $y = 0$, $x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$. (08 Marks)

- b. Using Divergence theorem evaluate $\int_S \vec{F} \cdot ds$, where $\vec{F} = 4x \, i - 2y^2 \, j + z^2 \, k$ and S is the surface

bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. (06 Marks)

- c. Show that the Geodesics on a plane are straight lines. (06 Marks)

- 10 a. Verify Stoke's theorem for the vector field $\vec{F} = (2x - y)\mathbf{i} - yz^2\mathbf{j} - y^2z\mathbf{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$ bounded by its projection on the xy - plane. (08 Marks)
- b. Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx} \left[\frac{\partial f}{\partial y'} \right] = 0$. (06 Marks)
- c. Find the Extremals of the functional $\int_{x_0}^{x_1} \frac{y'^2}{x^3} dx$. (06 Marks)
