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First Semester B.E. Degree Examination, July/August 2021
Engineering Mathematics - I

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions.***

1 a. Find the n^{th} derivative of $y = \sin^2 x \sinh^2 x + \log_{10}(x^2 - 3x + 2)$. **(07 Marks)**

b. Find the Pedal equation for the curve $r = a + b\cos\theta$. **(06 Marks)**

c. Obtain radius of curvature for the parametric curve, $x = a(t - \sin t)$ $y = a(1 - \cos t)$. **(07 Marks)**

2 a. If $y = \tan^{-1}x$, prove that $(1 + x^2)y_{n+2} + 2(n + 1)xy_{n+1} + n(n + 1)y_n = 0$. Hence obtain $y_n(0)$. **(07 Marks)**

b. Find the Angle of intersection between the curves $r = 2\sin\theta$; $r = 2(\sin\theta + \cos\theta)$. **(06 Marks)**

c. Find the radius of curvature for the polar curve $r^2 = a^2\cos 2\theta$. **(07 Marks)**

3 a. Expand $\log(1+x)$ using Maclaurin's series upto the term containing x^4. **(07 Marks)**

b. State and prove Euler's theorem for homogeneous function of degree n. **(06 Marks)**

c. If $u = x + y + z,\ v = y + z,\ z = uvw$, find $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$. **(07 Marks)**

4 a. i) Evaluate $\lim\limits_{x\to 0}\left(\dfrac{1}{x} - \dfrac{1}{e^x - 1}\right)$ ii) Evaluate $\lim\limits_{x\to 0}\left(\dfrac{a^x + b^x}{2}\right)^{\frac{1}{x}}$. **(06 Marks)**

b. If $u = \sin^{-1}\left(\dfrac{x^3 + y^3}{x + y}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 2\tan u$. **(07 Marks)**

c. If $u = f(x - y, y - z, z - x)$, then prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

5 a. A particle moves along $x = t^3 - 4t$, $y = t^2 + 4t$, $z = 8t^2 - 3t^3$, where 't' denotes time. Find the magnitudes of velocity and acceleration at time $t = 2$. **(07 Marks)**

b. Assuming the validity of differentiation under integral sign prove that

$$\int_0^\infty e^{-x^2}\cos\alpha x\,dx = \frac{\sqrt{\pi}}{2}e^{-\alpha^2/4}.$$ **(07 Marks)**

c. Trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$, using general rules of tracing the curve. **(06 Marks)**

6 a. If $\vec{F} = \text{grad}\,(x^3 + y^3 + z^3 - 3xyz)$ find curl $\vec{F}$. Is $\vec{F}$ irrotational? **(07 Marks)**

b. Prove that if $\vec{F}$ is a vector point function div (curl $\vec{F}$) = 0. **(07 Marks)**

c. If $\vec{r}$ is a position vector of a point in space obtain div $\vec{r}$ and curl $\vec{r}$. **(06 Marks)**

7 a. Obtain the reduction formula for $\int \sin^m x \cos^n x dx$, where m and n are positive integers. **(06 Marks)**

b. Solve: $xy(1+xy^2)\frac{dy}{dx}=1$. **(07 Marks)**

c. Find the Orthogonal trajectories of a system of confocal and coaxial parabolas $y^2=4a(x+a)$. **(07 Marks)**

8 a. Solve: $(xy^3+y)dx+2(x^2y^2+x+y^4)dy=0$. **(07 Marks)**

b. Evaluate $\int_0^a \frac{x^7}{\sqrt{a^2-x^2}}dx$, using reduction formulae. **(06 Marks)**

c. Water at temperature 100°C cools in 10 minutes to 88°C in a room of temperature 25°C. Find the temperature of water after 20 minutes. **(07 Marks)**

9 a. Solve the following system of equations: $x+y+z=9$, $x-2y+3z=8$, $2x+y-z=3$, by Gauss Elimination method. **(06 Marks)**

b. Diagonalize the matrix $\begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & -1 \\ 0 & 0 & -3 \end{bmatrix}$. **(07 Marks)**

c. Find the largest Eigen value and the corresponding Eigen vector of the matrix, $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

Taking $[1 \quad 0 \quad 0]^T$ as the initial Eigen vector carryout six iterations. **(07 Marks)**

10 a. Solve the following system by LU – Decomposition method.
$x+y+z=1$, $3x+y-3z=5$, $x-2y-5z=10$. **(08 Marks)**

b. Find the Inverse transformation of :
$y_1 = 4x_1 + 6x_2 + 6x_3$
$y_2 = x_1 + 3x_2 + 2x_3$
$y_3 = -x_1 - 4x_2 - 3x_3$. **(06 Marks)**

c. Reduce the quadratic form,
$3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to the Canonical form. **(06 Marks)**

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