

15EC54

Fifth Semester B.E. Degree Examination, July/August 2021 Information Theory and Coding

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions.

- 1 a. Define Self Information, Energy, Rate of Information. (03 Marks)
 - b. A single TV picture is viewed as an array of black, white and grey dots with roughly 500 rows and 600 columns. It is assumed that these dots take one of the 10 brightness levels. Find the information conveyed by one picture. (04 Marks)
 - c. Prove Extremal property of Entropy using Total derivative formula. (09 Marks)
- 2 a. Prove that $H_s(S^m) = m_sH(s)$ bits/sym. (06 Marks)
 - b. Students graduating from an Engineering department this year shows the following tendency i) Some go abroad ii) Some join MNCs in India iii) Remaining join PG course. The tendency in the next year is given below.
 - i) 50% of those who went abroad will return back to India, out of which 80% would join MNCs in India and remaining take PG course.
 - ii) Among those who remain in India 80% go abroad.
 - iii) Those who had remained in India have not swapped their fields.

Based on the information given above write the suitable model and determine entropy of the source.

(10 Marks)

- 3 a. What do you mean by Source Encoding? List its properties. (04 Marks)
 - b. A DMS produces six symbols with probabilities of occurances $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{24}$, $\frac{1}{3}$, $\frac{1}{24}$. Encode the symbols using Shannon Fano algorithms. Compute Average code work length and efficiency. (05 Marks)
 - c. Consider a statistically independent value whose source alphabet $S = \{S_0, S_1, S_2, S_3, S_4, S_5\}$ with $P = \{0.5, 0.25, 0.125, 0.0625, 0.03125, 0.03125\}$. Using Shannon Encoding Algorithm , find Code words and compute Minimum average length, Efficiency and Variance. (07 Marks)
- 4 a. Prove Source Coding theorem. (07 Mark
 - b. A DMS has an alphabet $S = \{S_1, S_2, S_3, S_4, S_5, S_6\}$ with $P = \left\{\frac{1}{3}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{12}, \frac{1}{12}\right\}$. Construct Huffman code by taking the code alphabet $X = \{0, 1, 2\}$. Find code efficiency and write decision tree.
 - c. Write an explanatory note on Lempel Ziv algorithm. (04 Marks)
 - 5 a. Compute all entropy functions and write the graphical model for channel given below:

$$P(xy) = \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0.1 & 0.3 & 0 & 0 \\ 0 & 0.05 & 0.1 & 0 \\ 0 & 0 & 0.05 & 0.1 \\ 0 & 0 & 0.05 & 0 \end{bmatrix}$$
 (07 Marks)

b. Show that I(X; Y) = H(X) - H(X|Y) bits/sym.

(04 Marks)

- Compute channel capacity for the channel given below: $P(Y/X) = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$.
- For the JPM given below, compute Data transmission rate, Channel capacity, Efficiency and Redundancy. $r_s = 1000 \text{ sym/sec}$

$$P(xy) = \begin{bmatrix} 0.05 & 0 & 0.2 & 0.05 \\ 0 & 0.1 & 0.1 & 0 \\ 0 & 0 & 0.2 & 0.1 \\ 0.05 & 0.05 & 0 & 0.1 \end{bmatrix}.$$
 (07 Marks)

b. Derive an expression for Differential Entropy.

(04 Marks)

- c. A black and white TV picture may be viewed as 3×10^5 pixel per frame with 10 distinct equi probable brightness levels. Assume that rate of transmission is 30 picture frames/second and SNR = 30db. Using Channel Capacity theorem, compute minimum bandwidth to support for error free transmission of Video signal. (05 Marks)
- a. Consider the (T, 4) Linear Block Code, whose generator matrix is 7

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & ; & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & ; & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & ; & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & ; & 0 & 1 & 1 \end{bmatrix}$$

- Find all code words ii) Find Parity check matrix 4
- iii) Find minimum Hamming weight and distance.

(04 Marks)

- Obtain the code word for the message (1 0 1 0) for a (7, 4) cyclic code with
 - $g(x) = 1 + x + x^3$. Use a four stage shift register for encoding.

(07 Marks)

- Write the decoding circuit for an (n, k) Linear Block Code and Decoding steps. (05 Marks)
- Design a Single error correcting Hamming code for a message of length 4. (04 Marks) 8
 - In a Linear Block Codes the syndrome is given by $S_1 = r_1 + r_2 + r_3 + r_5$, $S_2 = r_1 + r_2 + r_4 + r_6$ $S_3 = r_1 + r_3 + r_4 + r_7$.
 - i) Find the parity check matrix.
 - ii) Write the syndrome computation circuit.
 - iii) What is the syndrome for the received data (1 0 1 1 0 1 1) and correct it. (06 Marks)
 - c. Design a syndrome calculator circuit for a (7,4) cyclic code having $g(x) = 1 + x + x^3$. Verify the circuit for receiving code vector $R = [1 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1]$. (06 Marks)
- Consider the (2, 1, 2) convolution encoder with $g^{(1)} = (111)$, $g^{(2)} = (101)$.
 - i) Find the constraint length ii) Find the rate iii) Draw the encoder block diagram
 - iv) Find the output of the message (1 0 0 1 1) using Time Domain Approach
 - v) Find the output of the message (1 0 0 1 1) using Transform Domain Approach.

(12 Marks)

b. Write short note on Golay codes.

(04 Marks)

- 10 a. For (2, 1, 3) Convolution encoder with $g_1 = (1 \ 0 \ 1 \ 1)$, $g_2 = (1 \ 1 \ 0 \ 1)$.
 - i) Draw State diagram ii) Draw Tree diagram iii) Draw Trellis diagram and Code word for the message (1 1 1 0 1). (12 Marks)
 - b. Write an explanatory note on Viterbi Algorithm.

(04 Marks)