
A COMPARISON OF HEMLER & LONGSTAFF MODEL AND HSU & WANG MODEL: THE CASE OF INDEX FUTURES

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ABSTRACT

The study investigates pricing performance of two alternative futures pricing models. The Hemler & Longstaff Model (HLM) and Hsu & Wang Model (HWM) (2004) for three futures indices of National Stock Exchange (NSE), India – CNX Nifty futures, Bank Nifty futures and CNX IT futures. The results shows that, the Hsu & Wang Model with an argument of real capital markets are imperfect and incomplete arbitrage mechanism provides much better pricing performance than the Hemler and Longstaff Model with stochastic interest rates and market volatility for all the three futures markets. CNX Nifty futures contract with highest trading history and trading volume has lowest pricing error than Bank Nifty futures and CNX IT futures Index for both the pricing models. This implies that degree of market imperfection has greater influence on reducing pricing errors of Indian futures markets than market volatility and stochastic interest rates. Therefore, investors should know the degree of market imperfection and average daily trading volume of the futures markets in which they would like to participate

Keywords: Futures pricing models, Hemler & Longstaff Model, Hsu & Wang Model, Pricing performance, Degree of Market Imperfection

1.INTRODUCTION

The growth and popularity of index futures market in India attracts financial academicians, Institutional investors, speculators and arbitrageur. One of the primary functions of futures markets is price discovery. A number of researchers have made an extensive effort to predict stock index futures price under various assumptions and economic conditions. Many researchers found a significant correlation between Index futures mispricing and Index volatility. Panayiotis C. Andreou and Yiannos A. Pierides (2008), Fung, Joseph K W; Draper, Paul (1999), Stephen P. Ferris, Gay and Jung (1999), Nai-fu chen, Charles J. Cunniff, and Robert A. Haugen (1995), John J. Merrick, Jr (1987). This claims that as market volatility increases, investors sell their underlying and futures positions

with relatively larger drops in futures prices. Thus, from the above discussion, stock market volatility seems to be one of the important factors in determining stock index futures prices. Motivated by these considerations Michael L. Hemler and Francis A. Longstaff (1991) followed the CIR (Cox *et al.*, 1985a,b) framework and developed a closed form general equilibrium model of stock index futures prices in a continuous economy with stochastic interest rate and market volatility. The implications of this general equilibrium model for stock index futures prices are tested using regression analysis. When the natural logarithm of the dividend adjusted futures to spot price ratio can be represented as linear function of two variables, the risk free interest rate and the market volatility, they find that market volatility has significant explanatory power.

Hsu- Wang (2004) argues HLM was developed under the assumption of perfect market but further, he states that capital markets are imperfect. First, index arbitrage involves transaction costs, including commissions, bid-ask spread, and taxes. Second, there are constraints on short sales and securities are not perfectly divisible. Third, price changes in securities and constant and continuous dividends cannot be expected always. Fourth, it's not always possible to purchase and sale exact number of the underlying index simultaneously. Fifth, there is a limitation on borrowing or lending at the same risk-free rate. Finally, traders may have asymmetric information. Further, Hsu- Wang (2004) includes the factor of price expectation (Expected growth rate) and uses an argument of the incomplete arbitrage mechanism and developed a pricing model of stock index futures in imperfect markets (here after Hsu- Wang model).

Many previous studies [Janchung Wang (2009) , Janchung Wang(2007) , , Janchung Wang & Hsinan Hsu (2005), Gay, Gerald D & Jung, Dae Y (Apr 1999), T.J. Brailsford and A.K Cusack (August 1997), Michael L. Hemler and Francis A. Longstaff (1991) and Bailey (1989)) compared Hemler and Longstaff Model with Cost of carry model. Janchung Wang & Hsinan Hsu (2006 a) , Janchung Wang & Hsinan Hsu (2006 b)] compared Hsu and Wang Model with Hemler & Longstaff Model and Cost of Carry Model with other pricing models. Motivated by the above considerations the present study undertaken (1) To compare pricing performance of Hsu and Wang model (HWM) and Hemler and Longstaff model (HLM). (2) To identify best pricing model based on pricing performance of futures indices.

1.1 Futures pricing Models:

Two alternative futures pricing models are compared in the present study. i.) Hemler and Longstaff Model (HLM) (1991) ii.) Hsu & Wang Model (HWM)(2004)

i.) Hemler and Longstaff model (1991)

$$L_t = \alpha + \beta_1 r_t + \beta_2 v_t + \varepsilon_t \quad (1)$$

Where $L_t = \ln (F_t e^{q\tau} / S_t)$ is the logarithm of the dividend adjusted futures / Spot price ratio, F_t is the theoretical Futures price, S_t is the underlying spot index, τ is the time to maturity (T-t) , r_t is the Risk free interest rate v_t is the market volatility α, β_1 & β_2 are the regression coefficients. ε is the error part assumed to be normally distributed with mean zero. The empirical testing of Hemler and Longstaff model involves two stage procedures. One , it is assumed that theoretical futures price derived from Hemler & Longstaff equilibrium model differ from actual or observed futures prices by a mean of zero . Hence the regression coefficients of α, β_1 & β_2 can be obtained. Second stage involves substituting the estimated α, β_1 & β_2 to the Hemler and Longstaff equilibrium model to generate the estimate of the dividend adjusted futures / Spot price ratio L_t . Finally the theoretical futures price (F_t) can obtain by inferring L_t .

ii.) Hsu & Wang Model (HWM).

Hsu & Wang (2004) incorporated price expectation parameter (u_α) and developed futures pricing model in imperfect market.

This study uses the following assumptions to derive a pricing model of stock index futures in imperfect markets: 1. the underlying stock index pays a continuous constant dividend yield, q , during the life of the futures contract. 2. The instantaneous degree of market imperfection remains constant throughout the life of the futures contract. 3. The underlying stock index price, S , follows a geometric Wiener process, as follows:

Hsu & Wang model considered a hedged portfolio that comprises one unit of spot index and x units of futures index. The model assumes that initially cash outflow is not required for the futures contract. Then the rate of return of the hedged portfolio is illustrated by

$$\frac{dP}{P} = (w_f u_f + u) dt + (w_f \sigma_f + \sigma) dZ \quad (2)$$

Where P is the hedged portfolio, $w_f = \frac{xF}{S}$, S represents the price of the underlying stock index, F denotes the price of the futures index, u & σ represents constant expected growth rate and constant volatility of the underlying stock index (S) respectively. u_f & σ_f denotes the instantaneous expected return on futures and instantaneous standard deviation of return on futures respectively and dZ is a geometric wiener process.

Further, If $w_f = -\frac{\sigma}{\sigma_f}$ then $w_f \sigma_f + \sigma = 0$. u_f & u remain same but second part in equation 1 become zero. It indicates that, the hedged portfolio (P) can expected certainly and hedged portfolio becomes riskless. However in order to keep this portfolio risk free, it's necessary to rebalance w_f continuously until expiration of the futures contract. Figlewski(1989) found that, forming riskless portfolio hedge and continuously rebalancing hedged positions is only possible in perfect markets. Because of incomplete arbitrage mechanism and arbitrage process is exposed to heavy risk, the hedged portfolio is not possible to riskless at any point of time. Let u_p & σ_p represents the instantaneous expected rate of return of the hedged portfolio (P) & the coefficient of Wiener process dZ in the equation 1 respectively. This can be obtained as follows.

$$w_f u_f + u = u_p \quad (3)$$

$$w_f \sigma_f + \sigma = \sigma_p \quad (4)$$

From equation 2 & 3 the result of partial differential equation can be obtained as follows

$$\frac{1}{2} \sigma^2 S^2 F_{ss} + u_\alpha S F_s + F_t = 0 \quad (5)$$

Where u_α is the Hsu & Wang's price expectation parameter $u_\alpha = [u_p - q] - (u - q) \frac{\sigma_p}{\sigma} / (1 - \frac{\sigma_p}{\sigma})$

The second order partial differential equation 4 along with the following futures index price terminal condition at expiry date (T), fully characterize the futures index price.

$$F(S, T) = S_t$$

Finally the solution of this PDE is given by

$$F(S, T) = S_t e^{u_\alpha(T-t)} \quad (6)$$

Equation (6) is known as Hsu & Wang Futures pricing Model.

1.2 History and Institutional background of all the three futures indices

Table 1: Descriptive Statistics of daily trading Volume for all the three futures indices

Descriptive Statistics of daily Volume				
Contract	N	Mean	Max	Min
CNX Nifty Futures	1741	442492.60	1338598	1935
Bank Nifty Futures	1741	52007.03	256601	7
CNXIT Futures	1741	305.26	3037	1

Source: Collected and Compiled by the Authors

National Stock Exchange (NSE) India, is country's leading stock exchange was incorporated in the year November 1992 and recognized as a stock exchange in April 1993. Currently about 1500 securities listed on NSE. Index value calculates based on Free Float market capitalization Method (After 2008). NSE futures contracts have a maximum of 3-month trading cycle - one month (near), the two month (next) and the three month (far). A new futures contract is introduced on the immediate next trading day of the expiry of the near month contract. The new contract will be introduced for three month duration. This way, at any point in time, there will be 3 contracts available for trading in the market. Nifty futures contracts mature on the last Thursday of every month. If the last Thursday of every month is happened to be a trading holiday, the contracts expire on immediate previous trading day. The futures contract is cash settle only.

Table 1 shows that the CNX Nifty Index futures contract are based on popular underlying index and market bench mark CNX Nifty Index, constitutes 50 major stocks and began trading on NSE on 12 June 2000. The Bank Nifty Index futures contract based on the underlying index of CNX Bank Nifty Index constitutes 12 stocks from the banking sector and began trading on June 2005. The CNXIT Index futures contract are based on the underlying index of CNXIT Index, constitutes 20 major stocks from IT sector which trade on the National Stock Exchange and began trading on august 2003. Average daily trading volume during the period of the study was 442492, 52007 and 305 contracts for of CNX Nifty futures, CNX Bank futures and CNX IT futures index respectively. The importance of CNX Nifty index, Bank Nifty Index and CNX IT Index cannot be under rated as it constitutes 66.85%, 15.55% and 11.27% of free float market capitalization of NSE respectively. Bank Nifty index and CNX IT index represent about 89.90% and 97.25% of the free float market capitalization of the stocks constituting part of the Banking sector and the IT sector as on June 30, 2014 respectively.

2. DATA AND METHODOLOGY

Indices are selected based on their trading history in NSE. In precise, the indices which were launched before April 1, 2007 are considered for the study. For the CNX Nifty futures, CNX IT futures and Bank Nifty futures contract, only near month (one month) contracts were considered for this study because the nearest maturity contracts have significant trading volume compares to next month (two months) & far month (three months) contracts. Daily closing prices were obtained for all the three futures indices for the period from 1 April 2007 to 31 March 2014. The study used equally weighted moving average of past spot index returns to estimate the variance of underlying index returns. The 364- day government of India Treasury bill rates were used as proxy for risk free interest rates and obtained from RBI database. Implied method is used to estimate price expectation parameter for Hsu & Wang model. Independent t test is used to test whether the MAPE statistics generated from each model is statistically different.

2.1 Hypothesis

H_0 = There is no significant difference in MAPE statistics generated from Hsu & Wang Model and Hemler and Longstaff Model.

2.2 Measuring the pricing performance for the two models

Following Hsu & Wang (2004), pricing performance between Hemler & Longstaff Model (HLM) and Hsu & Wang Model (HWM) can be measured by calculating the mean absolute error (MAE), the mean percentage error (MPE) and mean absolute percentage error (MAPE) are illustrated as follows.

$$MAE = \frac{1}{N} \sum_{t=1}^N |AF_t - F_t| \quad (7) \quad MPE = \frac{1}{N} \sum_{t=1}^N \frac{AF_t - F_t}{AF_t} \times 100 \quad (8)$$

$$MAPE = \frac{1}{N} \sum_{t=1}^N \left| \frac{AF_t - F_t}{AF_t} \right| \times 100 \quad (9)$$

Where AF_t is the actual price of stock index futures at time t and F_t is the theoretical price of stock index futures at time t . Further, to compare the futures pricing error statistics between Hsu & Wang Model (HWM) and Hemler and Longstaff model (1991) t - test was used to test whether the MAPE statistics obtained from two pricing models were significantly different.

2.3 Parameter estimation of the Hemler and Longstaff model

Volatility of the underlying index returns (V_t) is the only parameter that cannot be directly observed in Hemler and Longstaff model. To estimate time varying volatility in underlying index returns, equally weighted moving average method is commonly employed by the estimators. Following Hsu & Wang (2004), The study used equally weighted moving average of past spot index returns to estimate the variance of underlying index returns.

$$V_{dt} = \frac{1}{N} \sum_{i=t-N}^{t-1} (R_i - \bar{R})^2 \quad (10)$$

$$R_i = \ln(S_i | S_{i-1}) \quad (11)$$

$$\bar{R} = \frac{1}{N} \sum_{i=t-N}^{t-1} R_i \quad (12)$$

Where V_{dt} is the variance of underlying index returns estimate on day t ; R_i is the spot index return on day i ; S_i is the spot index price on day i ; \bar{R} denotes the mean return of spot index; and n is the length of the period set to a value of 20 days, as suggested by Chiras and Manaster (1978). The variance of underlying index returns per annum (V_t) should be calculated from the variance per trading day V_{dt} using the formula.

$$V_t = V_{dt} \times (\text{Number of trading days per annum}) \quad (13)$$

2.4 Estimation of Price expectation parameter for Hsu & Wang model:

Implied method: For Hsu & Wang Model in imperfect markets, only price expectation parameter (u_{α}) cannot be estimated directly. The spot index that pays constant dividend yield, the implied $u_{\alpha, t-1}$ can be obtained from eq 28 of Hsu and Wang model (2004).

$$u_{\alpha, t-1} = \frac{1}{T-(t-1)} \ln \frac{F_{t-1}}{S_{t-1}} \quad (14)$$

Table 2: Regression Results

Futures Index	N	A	β_1	β_2	R ²	F	DW
NIFTY	1703	-0.0012*** (0.005)	-.005*** (0.000)	.044*** (0.000)	0.057	51.793*** (0.000)	0.536
BANK	1703	-0.003*** (0.000)	0.064*** (0.000)	-0.014 *** (0.026)	0.059	53.108*** (0.000)	0.557
IT	1703	0.0000 (0.897)	0.024 *** (0.001)	-0.05*** (0.000)	0.024	21.28*** (0.000)	0.897

Note. *** Significant at the 1 % Level. Source: Collected and Compiled by the Authors

Table 2 summarizes the results of the linear regression model given in expression (2). For all the three futures markets the F statistics results are significant at 1% level. Regression results supports that risk free Interest rates and Market Volatility significantly impact the natural logarithm of the dividend – adjusted futures to spot ratio.

3. EMPIRICAL RESULTS

Table 3: Pricing Performance of both the models for all the three futures indices

Futures Index	Model	N	Absolute Error		Percentage error		Absolute Percentage error	
			Mean (%)	SD (%)	Mean (%)	SD (%)	Mean (%)	SD (%)
NIFTY	HLM	1703	12.0092	10.3505	-0.0243	0.3316	0.2440	0.2258
	HWM	1740	7.9264	7.70143	0.0093	0.2262	0.1611	0.1589
BANK	HLM	1703	25.0662	23.3729	0.0054	0.3620	0.2701	0.2410
	HWM	1740	15.8919	15.2893	0.0088	0.2505	0.1811	0.1733
CNX IT	HLM	1703	67.0291	64.4995	-0.0298	1.8954	1.3148	1.3652
	HWM	1740	10.6948	11.4074	0.0075	0.3357	0.2032	0.2673

Note: OP- Over Price, UP – Under Price; OP= -ve (Ft > AF), UP = +ve ; Ft < AF

Table 4: Results of statistical tests for difference in MAPE between the futures pricing models

Futures Index	Pricing Models	N	t- value	Sig (2- tailed)
CNX NIFTY	HLM vs HWM	1703 - 1740	57.018***	0.000
BANK NIFTY	HLM vs HWM	1703 - 1740	89.049***	0.000
CNX IT	HLM vs HWM	1703 - 1740	217.247***	0.000

Note: *** Significant at the 1 % Level.

Pricing performance of HLM & HWM for all the three futures indices

According to table 3, the percentage error, HLM overprices two futures indices – Nifty futures index and IT futures index by an average of -0.0243% & -0.0298% respectively. Additionally, HLM under prices Bank nifty futures by an average of 0.0054%. Further, HWM under prices all the three futures indices Nifty futures, Bank nifty futures and IT futures index by an average of 0.0093%, 0.0088% & 0.0075% respectively. The MAPE of HWM for CNX Nifty futures, Bank Nifty futures and CNX IT futures is 0.1611%, 0.1811%, & 0.2032% respectively and lower than MAPE of HLM for the entire three futures indices. Overall, on the basis of mean percentage error (MPE) & MAPE, the best model preferred is HWM than HLM. This result supports Hsu & Wang (2006). The pricing performance of CNX Nifty futures contract is significantly better than that of Bank Nifty futures and CNX IT futures contract for both the pricing models. CNX Nifty futures contract with highest trading history and average trading volume has smallest pricing errors

than Bank Nifty futures and CNX IT futures. Additionally, CNX IT futures index with lowest average daily trading volume has highest pricing error for both HWM and HLM. Moreover the pricing error of HLM for CNX IT futures index is in higher magnitude than the rest. The MAPE of pricing models from the table 3 clearly shows that Hsu & Wang Model (HWM) incorporating degree of market imperfection and Price expectation parameter outperforms Hemler & Longstaff Model (HLM) incorporating market volatility and stochastic interest rate. Table 4 shows the result of Independent t test. Independent t test is used to test whether the MAPE statistics generated from each model is significantly different. For all the three futures indices – CNX Nifty, Bank nifty and CNX IT futures index the table clearly indicates that the MAPE statistics generated from each model is statistically significant at 1 %.

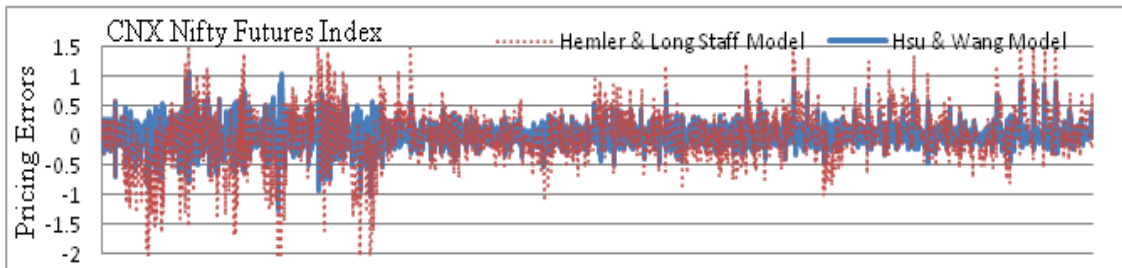


Fig1: Percentage pricing errors of HLM & HWM for CNX Nifty Futures Index

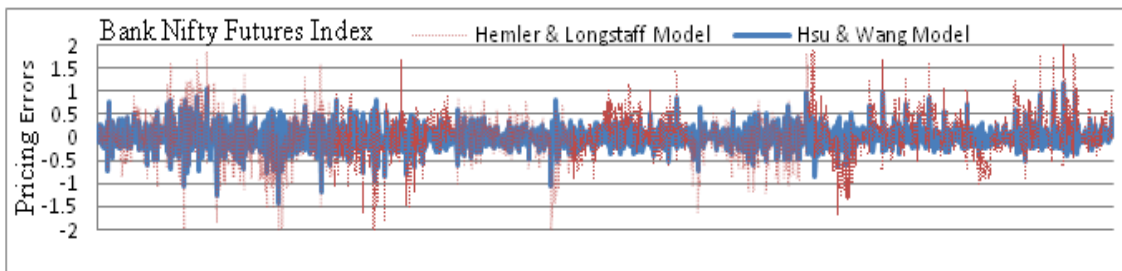


Fig 2: Percentage pricing errors of HLM & HWM for Bank Nifty Futures Index



Fig 3: Percentage pricing errors of HLM & HWM for CNX IT Futures Index

Figures 1 to 3 plot the percentage errors of Hsu & Wang Model and Hemler & Longstaff Model for CNX Nifty futures index, Bank Nifty futures index and CNX IT futures index respectively. It clearly shows that Percentage errors of the Hemler and Longstaff Model much higher than Hsu & Wang Model for all the three futures markets. Further Fig 3 clearly shows that percentage errors of Hemler & Longstaff Model for CNX IT Futures market are higher in magnitude than Hsu & Wang Model.

4. CONCLUSION

In terms of performance of pricing models, the Hsu & Wang Model outperforms Hemler & Longstaff Model and provides lowest MAPE than HLM for all the three futures Indices. HLM with market volatility and stochastic interest rates totally failed to provide better pricing performance than HWM. This implies that degree of market imperfection and price expectation parameter might influence to reduce the pricing error than Market volatility and stochastic interest rates. In terms of pricing performance of futures indices, CNX Nifty futures index with highest trading history and trading volume is preferred than Bank Nifty Futures index and CNX IT futures index for both the pricing models. Moreover the pricing error of HLM for CNX IT futures index is in higher magnitude than the rest. This implies that average daily trading volume of indices might influence pricing errors. Therefore, investors should know the degree of market imperfection, average daily trading volume of the futures markets in which they would like to participate. The study suggests (1) investigating of degree of market imperfection derived by Hsu and Wang (2004) and its impact on pricing performance of Indian futures markets. (3) Estimation of Hsu & Wang (2004) price expectation parameter (u_a) and Market volatility (V_t) by developing other efficient methodologies and assess the pricing performance of Hsu & Wang Model and Hemeler & Long staff model for Indian futures market respectively.

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