USN

Third Semester B.E. Degree Examination, Jan./Feb. 2021
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: Answer any FIVE full questions, selecting at least TWO questions from each part.

PART – A

1 a. If $f(x) = |\cos x|$, expand f(x) as a Fourier series in the interval $(-\pi, \pi)$. **(06 Marks)**

b. Obtain a half range Fourier cosine series for $x \sin x$ in the interval $(0, \pi)$. Hence show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ldots\infty = \frac{\pi - 2}{4}$. **(07 Marks)**

c. The following table gives the variations of periodic current over a period.

t sec	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
A amp	1.98	1.30	1.05	1.30	–0.88	–0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic. **(07 Marks)**

2 a. Find the Fourier transform of $f(x) = \begin{cases} 1 \text{ for } & |x| < 1 \\ 0 \text{ for } & |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(06 Marks)**

b. Find the Fourier sine transform of $e^{-|x|}$, hence evaluate $\int_0^\infty \frac{x \sin mx}{1 + x^2} dx$. **(07 Marks)**

c. Find the Fourier transform of $f(x) = e^{-a^2}$. **(07 Marks)**

3 a. A tightly stretched string with fixed end points x = 0, and $x = \ell$ is initially in a position given by $y = y_0 \sin^3(\pi x / \ell)$. If it is released form rest from this position, find the displacement y(x, t). **(06 Marks)**

b. Solve the differential equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ for the conduction of heat along a rod without radiation, subject to the following conditions:

(i) u is not infinite for $t \to \infty$ (ii) $\frac{\partial u}{\partial x} = 0$ for x = 0 and $x = \ell$

(iii) $u = \ell x - x^2$ for t = 0 between x = 0 and $x = \ell$. **(07 Marks)**

c. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions, u(0, y) = 0, u(π, y) = 0, u(x, ∞) = 0 and u(x, 0) = K sin 2x. **(07 Marks)**

4 a. Fit a parabola $y = a + bx + cx^2$ to the following data: **(06 Marks)**

x	–2	–1	0	1	2
y	–3.150	–1.390	0.620	2.880	5.378

b. Use the Graphical method to minimize, $Z = 20x + 10y$, subject to the constraints, $x + 2y \le 40$, $3x + y \ge 30$, $4x + 3y \ge 60$, $x \ge 0$, $y \ge 0$. **(07 Marks)**

c. Use the Simplex method to maximize, $Z = 3x + 4y$, subject to the constraints $2x + y \le 40$, $2x + 5y \le 180$, $x \ge 0$, $y \ge 0$. **(07 Marks)**

PART – B

5 a. Using Newton Raphson method, find a real root of $x \sin x + \cos x = 0$ which is near $x = \pi$, correct to four decimal places. **(06 Marks)**

b. Solve the system of equations by relaxation method $x + y + 54z = 110$, $27x + 6y - z = 85$, $6x + 15y + 2z = 72$. **(07 Marks)**

c. Using Rayleigh's power method, find largest eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$ taking initial vector as $[1, 0.8, -0.8]^T$. **(07 Marks)**

6 a. Estimate the values of f(22) and f(42) from the following table:

x	20	25	30	35	40	45
y = f(x)	354	332	291	260	231	204

(07 Marks)

b. Use Lagrange's formula to find the polynomial f(x), given:

x	0	2	3	6
y = f(x)	648	704	729	792

(07 Marks)

c. Use Simpson's 1/3 rule to evaluate $\int_0^5 \frac{dx}{4x+5}$, dividing the range into 10 sub intervals. **(06 Marks)**

7 a. Solve $25u_{xx} = u_{tt}$ at the pivotal points given $u(0,t) = 0 = u(5,t)$; $u_t(x, 0) = 0$ and $u(x,0) = \begin{cases} 20x, & 0 \le x \le 1 \\ 5(5-x), & 1 \le x \le 5 \end{cases}$ by taking h = 1, compute u(x, t) for $0 \le t \le 1$. **(07 Marks)**

b. Solve $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t}$ subject to the conditions u(0, t) = 0 = u(4, t) and u(x, 0) = x(4 – x) by taking h = 1. Find the values upto t = 5. **(07 Marks)**

c. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in the following square region with the boundary conditions as indicated in the Fig.Q7(c).

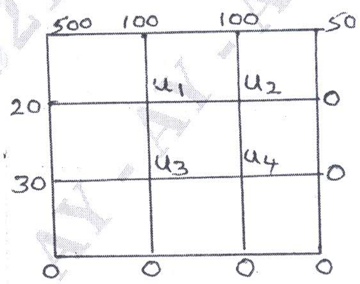

Fig.Q7(c) **(06 Marks)**

8 a. If $\bar{u}(z) = \frac{2z^2 + 3z + 12}{(z-1)^4}$, find the values of u_0, u_1, u_2. **(07 Marks)**

b. Obtain the inverse z transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$. **(06 Marks)**

c. Solve the difference equation, $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using z-transform. **(07 Marks)**

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