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14MAT21

Second Semester B.E. Degree Examination, Jan./Feb. 2021
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Solve $\frac{d^2y}{dx^2} - \frac{3dy}{dx} + 2y = xe^{3x} + \cos 2x$. **(07 Marks)**

b. Solve $\frac{d^2y}{dx^2} - \frac{2dy}{dx} + 10y = 0$, with $y(0) = 4$, $y'(0) = 1$. **(06 Marks)**

c. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2} - \frac{3dy}{dx} + 2y = x^2 + e^x$. **(07 Marks)**

2 a. Solve $\frac{d^4y}{dx^4} + \frac{8d^2y}{dx^2} + 16y = 0$. **(06 Marks)**

b. Solve $(D + 2)(D - 1)^2y = e^{-2x} + 2\sin x$ **(07 Marks)**

c. Solve by the method of variation of parameters $\frac{d^2y}{dx^2} - \frac{2dy}{dx} + 2y = e^x \tan x$. **(07 Marks)**

Module-2

3 a. Solve $\frac{dx}{dt} - 2y = \cos 2t; \frac{dy}{dt} + 2x = \sin 2t$. **(06 Marks)**

b. Solve $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8y = 65\cos(\log x)$. **(07 Marks)**

c. Solve $x^2p^2 + 3xyp + 2y^2 = 0$. **(07 Marks)**

4 a. Solve $y = 2px + \tan^{-1}(xp^2)$. **(06 Marks)**

b. Solve $(2x + 1)^2 \frac{d^2y}{dx^2} - 2(2x + 1)\frac{dy}{dx} - 12y = 6x$. **(07 Marks)**

c. Solve $p = \sin(y - xp)$. **(07 Marks)**

Module-3

5 a. Form the partial differential equation by eliminating the arbitrary functions form $z = f(x + at) + g(x - at)$. **(06 Marks)**

b. Solve the equation $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ given that $\frac{\partial z}{\partial y} = -2\sin y$. When $x = 0$ and $z = 0$ when y is odd multiple of $\frac{\pi}{2}$. **(07 Marks)**

c. Evaluate $\iint_R xy\,dx\,dy$, where R is the region bounded by the x-axis, the ordinate $x = 2a$ and the parablola $x^2 = 4ay$, $a > 0$. **(07 Marks)**

6 a. Derive one dimensional heat equation. **(06 Marks)**

b. By changing the order of integration, evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dy \, dx .$$ **(07 Marks)**

c. Evaluate $\int_0^{\pi/2} \int_0^{a\sin\theta} \int_0^{\frac{a^2-r^2}{a}} r \, dz \, dr \, d\theta$ **(07 Marks)**

Module-4

7 a. Using double integration, find the area lying between the parabola $y = 4x - x^2$ and the line $y = x$. **(06 Marks)**

b. Prove that $\int_0^{\pi/2} \sqrt{\sin\theta d\theta} \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$. **(07 Marks)**

c. Show that the vector field $F = (\cos\theta + \sin\theta)e_r + (\cos\theta - \sin\theta)e_\theta + e_z$, given in cylindrical polar coordinates, is solenoidal. **(07 Marks)**

8 a. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(06 Marks)**

b. Find the curt of the vector field $f = (r^2 \cos\theta)\, e^r - \frac{1}{r} e_\theta + \frac{1}{r\sin\theta} e\phi$, given on spherical polar coordinates. Also determine $f \cdot \text{curl} f$. **(07 Marks)**

c. Show that the vector field $f = (r^2 \sin 2\theta)e_r + (r^2 \cos 2\theta)e_\theta + \frac{1}{2}(r^2 \sin 2\theta)\, e_z$. **(07 Marks)**

Module-5

9 a. Find the Laplace transform of $2^t + \frac{\cos 2t - \cos 3t}{t} + t\sin t$. **(06 Marks)**

b. A Periodic function of period 2a is defined by

$$f(t) = \begin{cases} t, & \text{if } 0 \le t \le a \\ 2a - t, & \text{if } a < t \le 2a \end{cases}$$ Show that $L\{f(t)\} = \frac{1}{s^2} \tan h \frac{(as)}{2}$. **(07 Marks)**

c. Find the inverse Laplace transform of $\frac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}$. **(07 Marks)**

10 a. Express the function $f(t) = \begin{cases} \cos t, & 0 < t \le \pi \\ \sin t, & t > \pi \end{cases}$

interms of unit step function and hence find its Laplace transform. **(06 Marks)**

b. Using convoluting Theorem. Evaluate $L^{-1}\left\{ \frac{s}{(s^2 + a^2)^2} \right\}$ **(07 Marks)**

c. Solve $\frac{d^2y}{dt^2} + \frac{5dy}{dt} + 6y = 5e^{2t}$ given that $y(0) = 2$, $\frac{dy(0)}{dt} = 1$ by using Laplace transform method. **(07 Marks)**

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