

CBCS SCHEME

17MAT11

First Semester B.E. Degree Examination, Jan./Feb. 2021 Engineering Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the nth derivative of

$$\frac{x}{(x-1)(2x+3)} \tag{06 Marks}$$

- b. Find the angle of intersection of the curves $r = \frac{a}{1 + \cos \theta}$ and $r = \frac{b}{1 \cos \theta}$. (07 Marks)
- c. Find the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1, 1). (07 Marks)

OR

- 2 a. If $y = e^{a \sin^{-1} x}$ prove that $(1-x^2)y_{n+2} (2n+1)xy_{n+1} (n^2 + a^2)y_n = 0$. (06 Marks)
 - b. Find the pedal equation of $r^2 = a^2 \sec 2\theta$. (07 Marks)
 - c. Find the radius of curvature of the curve $r^n = a^n \sin n\theta$. (07 Marks)

Module-2

- 3 a. Obtain the Taylor's expansion of tanx about $x = \frac{\pi}{4}$ upto third degree terms. (06 Marks)
 - b. Evaluate $\lim_{x \to 0} \left(\frac{2^x + 3^x + 4^x}{3} \right)^{1/x}$ (07 Marks)
 - c. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$ (07 Marks)

OR

- 4 a. If $u = \sin^{-1}\left(\frac{x^2y^2}{x+y}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$. (06 Marks)
 - b. Obtain the Maclaurin's expansion of log(sec x) upto fourth degree terms. (07 Marks)
 - c. If x + y + z = u, y + z = v, z = uvw find the Jacobian $J\left(\frac{x, y, z}{u, v, w}\right)$ (07 Marks)

Module-3

- 5 a. If $\phi = x^2 + y^2 + z^2$, $\vec{F} = x^2 \hat{i} + y^2 \hat{j} + z^2 \hat{k}$ then find grad ϕ , div \vec{F} and curl \vec{F} . (06 Marks)
 - b. A particle moves on the curve $x = 2t^2$, $y = t^2 4t$, z = 3t 5 where t is the time. Find the components of velocity and acceleration at t = 1 in the direction of the vector i 3j + 2k.

 (07 Marks)
 - c. Prove that $\operatorname{curl}(\phi \vec{A}) = \phi \operatorname{curl} \vec{A} + (\operatorname{grad} \phi \times \vec{A})$ (07 Marks)

Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at (2, -1, 2)

b. Show that the vector field $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ is irrotational and find ϕ such that $\vec{F} = \nabla \phi$. (07 Marks)

c. Prove that $\operatorname{div}(\phi \vec{A}) = \phi \operatorname{div} \vec{A} + (\operatorname{grad} \phi \cdot \vec{A})$

(07 Marks)

Module-4

Obtain the reduction formula for 7

$$\int_{0}^{\pi/2} \cos^{n} x \, dx \tag{06 Marks}$$

b. Solve $\frac{dy}{dx} = xy^3 - xy$ (07 Marks)

c. A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of the air being 40°C. What will be the temperature of the body after 40 minutes from the original? (07 Marks)

OR

a. Evaluate $\int_{0}^{2a} \frac{x^2}{\sqrt{2ax-x^2}} dx$ (06 Marks)

b. Solve $(x^2 + y^2 + x)dx + xy dy = 0$ (07 Marks)

c. Obtain the orthogonal trajectories of the family of curves $r^n = a \sin n\theta$. (07 Marks)

Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$$
b. Diagonalize the matrix $\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ (07 Marks)

(07 Marks)

c. Using Rayleigh's power method find the largest eigen value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$
 taking $(1 \ 1 \ 1)^T$ as the initial eigen vector. (07 Marks)

OR

10 a. Using Gauss-Siedel method, solve

$$20x + y - 2z = 17$$

$$3x + 20y - z = -18$$

$$2x - 3y + 20z = 25$$

using (0, 0, 0) as the initial approximation to the solution.

(06 Marks)

Show that the linear transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular and find the inverse transformation. (07 Marks)

c. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into the canonical form.

(07 Marks)