USN

Third Semester B.E. Degree Examination, Jan./Feb. 2021
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Note: Answer any FIVE full questions.***

1 a. Express the complex number $\dfrac{(3+2i)^2}{(4-3i)}$ in the form of x + iy. **(06 Marks)**

b. Find the modulus and amplitude of $(\sqrt{3}+i)$ and express it in polar form. **(07 Marks)**

c. Show that the real part of $\dfrac{1}{1+\cos\theta+i\sin\theta}$ is $\dfrac{1}{2}$. **(07 Marks)**

2 a. Obtain the n^{th} derivative of $e^{ax}\sin(bx+c)$. **(06 Marks)**

b. Find the n^{th} derivative of $\dfrac{4x}{(x-1)^2(x+1)}$. **(07 Marks)**

c. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(07 Marks)**

3 a. Find the pedal equation to the curve $r^2 \sin 2\theta = a^2$. **(06 Marks)**

b. Find the angle of intersection for the pair of curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$. Are they orthogonal? **(07 Marks)**

c. Using Maclaurin's series, expand $\tan x$ upto the term containing x^5. **(07 Marks)**

4 a. If $u = \dfrac{x+y}{x-y}$, verify that $\dfrac{\partial^2 u}{\partial x\partial y} = \dfrac{\partial^2 u}{\partial y\partial x}$. **(06 Marks)**

b. State Euler's theorem. If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$. **(07 Marks)**

c. If $u = \dfrac{yz}{x}$, $v = \dfrac{zx}{y}$ and $w = \dfrac{xy}{z}$, show that $J\left(\dfrac{u,v,w}{x,y,z}\right) = 4$. **(07 Marks)**

5 a. Derive the reduction formula for $\int \cos^n x\,dx$ where n is a +ve integer. **(06 Marks)**

b. Evaluate $\displaystyle\int_0^5 \int_0^{x^2} x(x^2+y^2)\,dx\,dy$ **(07 Marks)**

c. Evaluate $\displaystyle\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z)\,dx\,dy\,dz$ **(07 Marks)**

6 a. Define Beta and Gamma functions. Show that $\Gamma(n) = \int_0^1 \left(\log\frac{1}{y}\right)^{n-1} dy, \ (n > 0)$. **(06 Marks)**

b. Show that $\beta(m, n) = 2\int_0^{\frac{\pi}{2}} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \, d\theta$ **(07 Marks)**

c. Express the integral $\int_0^{\infty} e^{-x^2} dx$ in terms of gamma function. **(07 Marks)**

7 a. Solve $(xy + x)dy + (xy + y)\,dx = 0$. **(06 Marks)**

b. Solve $x(y - x)\dfrac{dy}{dx} = y(y + x)$. **(07 Marks)**

c. Solve $\dfrac{dy}{dx} + \dfrac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$. **(07 Marks)**

8 a. Solve $(D^4 + 2D^2 + 1)y = 0$. **(06 Marks)**

b. Solve $(D^3 - D + 6)y = e^{4x}$. **(07 Marks)**

c. Solve $(D^2 + 4)y = \cos 2x$. **(07 Marks)**

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