

#### 15MATDIP41

# Fourth Semester B.E. Degree Examination, Jan./Feb. 2021 Additional Mathematics – II

Time: 3 hrs.

BANGAL

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

## Module-1

1 a. Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$$

by applying elementary row transformations.

(06 Marks)

b. Solve the system of equations by Gauss-elimination method:

$$x + y + z = 6$$

$$x - y + 2z = 5$$

$$3x + y + z = 8$$

(05 Marks)

c. Find all eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

(05 Marks)

OR

2 a. Find all eigen values and all eigen vectors of the matrix

$$A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$
 (06 Marks)

b. Solve by Gauss elimination method:

$$3x + y + 2z = 3$$

$$2x - 3y - z = -3$$

$$x + 2y + z = 4$$

(05 Marks)

c. Find the inverse of the matrix 
$$\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$$
 using Cayley-Hamilton theorem.

(05 Marks)

#### Module-2

3 a. Solve 
$$(D^3 - 6D^2 + 11D - 6)y = 0$$

(06 Marks)

b. Solve 
$$(D^2 + 6D + 9)y = 2e^{-3x}$$

(05 Marks)

c. Solve by the method of variation of parameters 
$$(D^2 + 1)y = \tan x$$
.

(05 Marks)

4 a. Solve 
$$(D^3 - 5D^2 + 8D - 4)y = 0$$

(06 Marks)

b. Solve 
$$(D^2 - 4D + 3)y = \cos 2x$$

(05 Marks)

c. Solve by the method of undetermined coefficients 
$$y'' - y' - 2y = 1 - 2x$$
.

(05 Marks)

## Module-3

a. Find Laplace transform of cos<sup>3</sup>at.

(06 Marks)

b. A periodic function of period 2a is defined by

$$f(t) = \begin{cases} E & \text{for } 0 \le t \le a \\ -E & \text{for } a \le t \le 2a \end{cases} \text{ where E is a constant. Find } L\{f(t)\}.$$
 (05 Marks)

b. A periodic function of periodic function of periodic function of periodic function of  $f(t) = \begin{cases} E & \text{for } 0 \le t \le a \\ -E & \text{for } a \le t \le 2a \end{cases}$  where  $F(t) = \begin{cases} \cos t, & t \le \pi \\ \sin t, & t > \pi \end{cases}$ in terms of unit step function and hence find its (05 Marks) Laplace transform.

6 a. Find 
$$L\left\{\frac{\cos at - \cos bt}{t}\right\}$$
 (06 Marks)

- (05 Marks) Find L{sintsin2tsin3t}
- c. Express the function  $f(t) = \begin{cases} t^2, \\ 4t, \end{cases}$ in terms of unit step function and hence find its (05 Marks) Laplace transform.

7 a. Find 
$$L^{-1} \left\{ \frac{2s+3}{s^3-6s^2+11s-6} \right\}$$
 (06 Marks)

- b. Find  $L^{-1}\left\{\cot^{-1}\left(\frac{s}{a}\right)\right\}$ (05 Marks)
- c. Using Laplace transform method, solve the initial value problem  $y'' + 5y' + 6y = 5e^{2t}$ , given (05 Marks) that y(0) = 2 and y'(0) = 1

8 a. Find 
$$L^{-1}\left\{\frac{s+2}{s^2-4s+13}\right\}$$
 (06 Marks)

- b. Find  $L^{-1}\left\{\log\left(\frac{s^2+1}{s(s+1)}\right)\right\}$ (05 Marks)
- c. Using Laplace transforms, solve the initial value problem  $y' + y = \sin t$ , given that y(0) = 0. (05 Marks)

#### Module-5

- For any two events A and B, prove that  $P(A \cup B) = P(A) + P(B) P(A \cap B)$ 
  - If A and B are any two events with  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{1}{3}$  and  $P(A \cap B) = \frac{1}{4}$ , find P(A/B), P(B/A),  $P(\overline{A}/\overline{B})$  and  $P(\overline{B}/\overline{A})$ (05 Marks)
  - From 6 positive and 8 negative numbers, 4 numbers are selected at random and are multiplied. What is the probability that the product is positive? (05 Marks)

#### OR

- State and prove Baye's theorem. (06 Marks) 10
  - A book shelf contains 20 books of which 12 are on electronics and 8 are on mathematics. If 3 books are selected at random, find the probability that all the 3 books are on the same subject.
  - c. The machines A, B, C produce 50%, 30% and 20% of the items in a factory. The percentage of defective outputs of these machines are 3, 4 and 5 respectively. If an item is selected at random is found to be defective, then determine the probability that the item was manufactured by machine A. (05 Marks)

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