CBCS SCHEME

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15CS36

Third Semester B.E. Degree Examination, Jan./Feb. 2021
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Define the following terms with an example : i) Conjunction ii) Tautology iii) Quantifiers iv) Proposition v) Conditional or Implication vi) Dual of statement. **(06 Marks)**

b. Prove the validity of the following arguments :

i)
$$\begin{array}{l} p \to r \\ \neg p \to q \\ q \to s \\ \hline \therefore \neg r \to s \end{array}$$

ii)
$$\begin{array}{l} (\neg p \vee \neg q) \to (r \wedge s) \\ r \to t \\ \neg t \\ \hline \therefore p \end{array}$$

(06 Marks)

c. Find the negation of each of the following quantified statements :

i) $\forall x, \forall y\ [(x > y) \to ((x - y) > 0)]$

ii) $\forall x, \exists y\ [(p(x, y) \wedge q(x, y)) \to r(x, y)]$. **(04 Marks)**

OR

2 a. Prove that the following compound propositions are tautologies :

i) $[p \wedge (p \to q)] \to q$ ii) $\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$. **(05 Marks)**

b. Prove the following by using laws of logic :

i) $[(p \to q) \wedge (\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg (q \vee p)$

ii) $[p \vee q \vee (\neg p \wedge \neg q \wedge r)] \Leftrightarrow (p \vee q \vee r)$. **(05 Marks)**

c. Show that "If n is an odd integer then n + 11 is an even integer" by i) Direct proof ii) An indirect proof iii) Proof by contradiction. **(06 Marks)**

Module-2

3 a. Prove the following by Mathematical Induction :

$$1^3 + 2^3 + 3^3 + \ldots.. + n^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

(05 Marks)

b. i) How many arrangements are possible for all the letters in the word SOCIOLOGICAL?
ii) In how many of these arrangements A & G are adjacent?
iii) In how many of these arrangements all the vowels are adjacent? **(06 Marks)**

c. A sequence $\{a_n\}$ is defined recursively by $a_1 = 4$, $a_n = a_{n-1} + n$ for $n \geq 2$. Find a_n in explicit form. **(05 Marks)**

OR

4 a. If Fi's are the Fibonacci numbers and Li's are the Lucas numbers, prove that $L_{n+4} - L_n = 5\,F_{n+2}$ for all integers $n \geq 0$. **(06 Marks)**

b. A certain college question paper contains 3 parts A, B and C with 4 questions in part A, 5 questions in part B & 6 questions in part C. It is required to answer 7 questions by selecting atleast 2 questions from each part. In how many different ways can a student solve the question paper? **(06 Marks)**

c. Find the coefficient of :
   i) $x^2 y^2 z^3$ in the expansion of $(x + y + z)^7$.
   ii) $v^2 w^3 x^2 y^5 z^4$ in the expansion of $(v + w + x + y + z)^{16}$. **(04 Marks)**

Module-3

5 a. Let A = {a, b, c, d} and B = {2, 4, 5, 7}. Determine the following :
   i) 1 A × B1.
   ii) Number of relations from A to B.
   iii) Number of relations from A to B that contain (a, 4) and (c, 7).
   iv) Number of relations from A to B that contain exactly six ordered pairs.
   v) Number of binary relations on A that contain atleast Fourteen ordered pairs. **(06 Marks)**

b. Let f, g, h be functions from z to z, define by $f(x) = x^2$, $g(x) = x + 5$ and $h(x) = \sqrt{x^2 + 2}$. Determine (h o (g o f)) (x) and ((h o g) o f) (x). Verify that h o (g o f) = (h o g) o f. **(05 Marks)**

c. Let A = {1, 2, 3, 4} and R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 1), (3, 3), (1, 3), (4, 1), (1, 4), (4, 4)}. Is R is an equivalence relation? Find the corresponding partition on A. **(05 Marks)**

OR

6 a. Prove that if f : A → B, g : B → C are invertible functions, the g o f : A → C is invertible and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. **(05 Marks)**

b. Let A = {1, 2, 3, 4} and R = {(1, 1), (1, 2), (2, 3), (3, 4)}, S = {(3, 1), (4, 4), (2, 4), (1, 4)} be relations on A. Determine the relations R o S, S o R, R^2 and S^2. Write down their matrices. **(05 Marks)**

c. Consider the Hasse diagram of a POSET (A, R) given in Fig. Q6(c).
   i) Determine the relation matrix R   ii) Construct the digraph for R
   iii) Write maximal, minimal, greatest and least elements. **(06 Marks)**

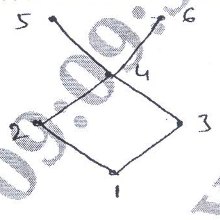

Fig. Q6(c)

Module-4

7 a. How many integers between 1 and 300 (inclusive) are i) divisible by atleast one of 5, 6, 8? ii) divisible by none of 5, 6, 8? **(05 Marks)**

b. Determine in how many ways can the letters in the work ARRANGEMENT be arranged so that there are exactly two pairs of consecutive identical letters. **(06 Marks)**

c. Find the Rook polynomial for 3 × 3 board by using the expansion formula. **(05 Marks)**

OR

8 a. A person invests Rs 100,000 at 12% interest compounded annually : i) Find the amount at the end of 1^{st}, 2^{nd}, 3^{rd} year ii) Write the general explicit formula iii) How long will it take to double the investment. **(06 Marks)**

b. Solve the recurrence relation $a_{n+2} - 8a_{n+1} + 16a_n = 8(5^n) + 6(4^n)$, where $n \geq 0$ and $a_0 = 12$, $a_1 = 5$. **(05 Marks)**

c. An apple, a banana, a mango and an orange are to be distributed to four boys B_1, B_2, B_3 and B_4. The Boys B_1 and B_2 do not wish to have apple, B_3 does not want banana or mango and B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased? **(05 Marks)**

Module-5

9 a. Define the following : i) Complete Graph ii) Bipartite Graph iii) Isolated Vertex iv) Regular Graph v) Subgraph. **(05 Marks)**

b. Let G = (V, E) be simple graph of order $|V| = n$ and size $|E| = m$. If G is a bipartite graph, prove that $4m \leq n^2$. **(05 Marks)**

c. Construct an optimal prefix code for the symbols a , o, p, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. **(06 Marks)**

OR

10 a. Prove the following :

i) A path with n vertices is of length n-1.

ii) If a cycle has n vertices, it has n edges.

iii) The degree of every vertex in a cycle is two. **(06 Marks)**

b. Define Isomorphism. Verify the two graphs are Isomorphic. (Refer Fig. 10(b(i),(ii))

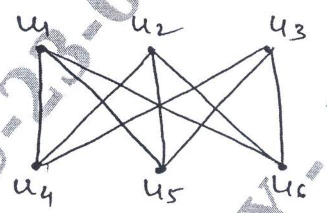

Fig. Q10(b (i))

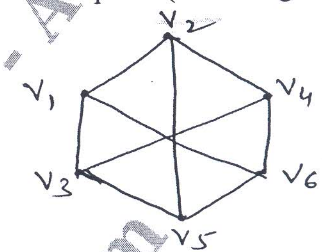

Fig. Q10(b (ii)) **(04 Marks)**

c. List the vertices in the tree given in Fig. Q10(c), when they are visited in :
i) Preorder ii) Postorder iii) Inorder Traversal. **(06 Marks)**

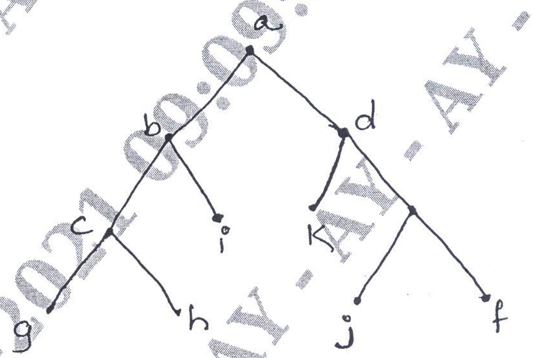

Fig. Q10(c)

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