CBCS SCHEME

USN

17MATDIP31

Third Semester B.E. Degree Examination, Aug./Sept. 2020
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the modulus and amplitude of, $1+\cos\alpha+i\sin\alpha$ (06 Marks)

b. Express the complex number $\dfrac{(1+i)(2+i)}{(3+i)}$ in the form a + ib. (07 Marks)

c. Find a unit vector normal to both the vectors $4i-j+3k$ and $-2i+j-2k$. Find also the sine of the angle between them. (07 Marks)

OR

2 a. Show that $\left[\dfrac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right]^n=\cos n\left(\dfrac{\pi}{2}-\theta\right)+i\sin n\left(\dfrac{\pi}{2}-\theta\right)$. (06 Marks)

b. If $\vec{A}=i-2j-3k$, $\vec{B}=2i+j-k$, $\vec{C}=i+3j-k$ find (i) $\left(\vec{A}\times\vec{B}\right)\times\left(\vec{B}\times\vec{C}\right)$ (ii) $\vec{A}\times\left(\vec{B}\times\vec{C}\right)$ (07 Marks)

c. Show that $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right]=\left[\vec{a},\vec{b},\vec{c}\right]^2$. (07 Marks)

Module-2

3 a. If $y=(x^2-1)^n$ then prove that $(1-x^2)y_{n+2}-2xy_{n+1}+n(n+1)y_n=0$. (06 Marks)

b. Find the pedal equation of the curve $r^m=a^m(\cos m\theta+\sin m\theta)$. (07 Marks)

c. Show that the following curves intersect orthogonally $r=a(1+\cos\theta)$, $r=b(1-\cos\theta)$. (07 Marks)

OR

4 a. Show that $\sqrt{1+\sin 2x}=1+x-\dfrac{x^2}{2}-\dfrac{x^3}{6}+\dfrac{x^4}{24}$...... using Maclaurin's series expansion. (06 Marks)

b. If $u=e^{ax+by}f(ax-by)$, prove that $b\dfrac{\partial u}{\partial x}+a\dfrac{\partial u}{\partial y}=2abu$. (07 Marks)

c. Find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ where $u=x^2+y^2+z^2$, $v=xy+yz+zx$, $w=x+y+z$. (07 Marks)

Module-3

5 a. Obtain a reducation formula for $\int\cos^n x\,dx$. (06 Marks)

b. Evaluate $\displaystyle\int_0^2\frac{x^4}{\sqrt{4-x^2}}dx$. (07 Marks)

c. Evaluate $\displaystyle\int_0^a\int_0^x\int_0^{x+y}e^{x+y+z}dz\,dy\,dx$. (07 Marks)

OR

6 a. Obtain a reducation formula for $\int \sin^n x \, dx$. **(06 Marks)**

b. Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} x^3 y \, dx dy$. **(07 Marks)**

c. Evaluate $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2) dz dy dx$. **(07 Marks)**

Module-4

7 a. A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$ and $z = 2t - 5$.
(i) Determine its velocity and acceleration.
(ii) Find the components of velocity and acceleration at t = 1 in the direction $2i + j + 2k$. **(06 Marks)**

b. Find the directional derivative of, $\phi = x^2yz + 4xz^2$ at (1, -2, -1) along $2i - j - 2k$. **(07 Marks)**

c. If $\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$ find a, b, c such that $\text{curl}\,\vec{F} = 0$ and then find ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

OR

8 a. If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$ prove that $\nabla(r^n) = nr^{n-2}.\vec{r}$ **(06 Marks)**

b. If $\vec{F} = (x + y + 1)i + j - (x + y)k$ show that $\vec{F}.\text{curl}\,\vec{F} = 0$. **(07 Marks)**

c. Show that $\vec{F} = (y + z)i + (z + x)j + (x + y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

Module-5

9 a. Solve : $\frac{dy}{dx} = \frac{y - x}{y + x}$. **(06 Marks)**

b. Solve : $(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$. **(07 Marks)**

c. Solve : $xy(1 + xy^2)\frac{dy}{dx} = 1$. **(07 Marks)**

OR

10 a. Solve : $\frac{dy}{dx} + y\cot x = \cos x$. **(06 Marks)**

b. Solve : $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$. **(07 Marks)**

c. Solve : $\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}$. **(07 Marks)**

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