CBCS SCHEME

USN | | | | | | | | | | |

15MAT41

Fourth Semester B.E. Degree Examination, Aug./Sept.2020
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Using the Taylor's series method, solve

$\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ at the point $x = 0.1$. Consider the series upto third degree terms. **(05 Marks)**

b. By using the modified Euler's method, solve $\frac{dy}{dx} = \log_e(x + y)$, $y(1) = 2$ at the point $x = 1.2$. Take $h = 0.2$ and carry out two modifications. **(05 Marks)**

c. Solve $\frac{dy}{dx} = x - y^2$ at $x = 0.8$, using Adams - Bashforth method, given that $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$. **(06 Marks)**

OR

2 a. Employ the Taylor's series method to find y(4.1) given that $\frac{dy}{dx} = \frac{1}{x^2 + y}$, $y(4) = 4$. Consider terms upto third degree. **(05 Marks)**

b. Given $\frac{dy}{dx} = 3e^x + 2y$ and $y(0) = 0$. Find y(0.1) using the Range-Kutta method. Take step length $h = 0.1$ **(05 Marks)**

c. Given $5x\frac{dy}{dx} + y^2 - 2 = 0$ and the set of values of (x, y) given in the following table, find y at $x = 4.5$ using the Milne's method.

x	4	4.1	4.2	4.3	4.4
y	1	1.0049	1.0097	1.0143	1.0187

(06 Marks)

Module-2

3 a. Given $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1$, $y'(0) = 0$, compute y(0.2) and y′(0.2) using fourth order Runge-Kutta method. **(05 Marks)**

b. Express $f(x) = x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials. **(05 Marks)**

c. If α and β are two distinct roots of $J_n(x) = 0$ then prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if $\alpha \neq \beta$. **(06 Marks)**

OR

4 a. Apply Milne's method to compute y(0.4) given that $y'' + xy' + y = 0$ and the table

x	0	0.1	0.2	0.3
y	1	0.995	0.9801	0.956
y′	0	-0.0995	-0.196	-0.2867

(05 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

b. Explain $J_{-\frac{1}{2}}(x)$ in terms if cos x. **(05 Marks)**

c. Derive Rodrigue's formula

$$P_n(x) = \frac{1}{2^n \cdot n!} \frac{d^n}{dx^n}\left[\left(x^2 - 1\right)^n\right]$$ **(06 Marks)**

Module-3

5 a. Define analytic function and obtain Cauchy – Riemann equations in Polar form. **(05 Marks)**

b. Evaluate $\int_c \frac{dz}{z^2 - 4}$ in the cases where 'C' is the circle |z+2| = 1 using Cauchy's integral formula. **(05 Marks)**

c. Discuss the transformation $w = z^2$. **(06 Marks)**

OR

6 a. Given $u = 3x^2y - y^3$. Find the analytic function f(z). **(05 Marks)**

b. Evaluate $\int_c \frac{e^z}{(z-1)(z-5)^2} dz$, where C is the circle $|z| = 8$ using Cauchy's Residue theorem. **(05 Marks)**

c. Find the bilinear transformation which maps the points z = 0, i, ∞ on to the points w = 1, –i, –1 respectively. **(06 Marks)**

Module-4

7 a. A random variable x has the following probability function for various values of 'x'

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

(i) Find k (ii) Evaluate $P(x < 6)$, $P(x \geq 6)$ and $P(3 < x \leq 6)$ **(05 Marks)**

b. Obtain Mean and Standard Deviation of the Exponential Distribution. **(05 Marks)**

c. A communication channel receives independent pulses at the rate of 12 pulses per micro second. The probability of transmission error is 0.001 for each micro second. Compute the probabilities of (i) no error during a micro second (ii) one error per micro second (iii) atleast one error per micro second (iv) two errors (v) atmost two errors. **(06 Marks)**

OR

8 a. The pen manufactured by a company will be defective is 0.1. If 12 such pens are selected at random, find the probability that
(i) exactly 2 will be defective
(ii) atleast 2 will be defective
(iii) none will be defective **(05 Marks)**

b. In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and S.D. of the distribution. (S.D = Standard deviations $\phi(0.5) = 0.1915$, $\phi(1.4) = 0.4192$). **(05 Marks)**

c. The joint probability distribution table for two random variables X and Y as follows:

X \ Y	–2	–1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine the marginal probability distributions of X and Y. Also compute (i) Expectations of X, Y and XY (ii) Covariance of X and Y. **(06 Marks)**

Module-5

9 a. Certain tubes manufactured by a company have mean life time of 800 hours and standard deviation of 60 hours. Find the probability that a random sample of 16 tubes taken from the group will have a mean life time
(i) between 790 hours and 810 hours
(ii) less than 785 hours
(iii) more than 820 hours
(iv) between 770 hours and 830 hours. **(05 Marks)**

b. A group of 10 boys fed on a diet A and another group of 8 boys fed on a different diet B for a period of 6 months recorded the following increase in weights (lbs).

Diet A:	5	6	8	1	12	4	3	9	6	10
Diet B:	2	3	6	8	10	1	2	8		

Test whether diets A and B differ significantly regarding their effect on increase in weight. ($t_{0.05}$ for 16 d.f = 2.12). **(05 Marks)**

c. Find the unique fixed probability vector for the regular stochastic matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$ **(06 Marks)**

OR

10 a. A random sample for 1000 workers in company has mean wage of Rs. 50 per day and standard deviation of Rs. 15. Another sample of 1500 workers from another company has mean wage of Rs. 45 per day and standard deviation of Rs. 20. Does the mean rate of wages varies between the two companies? Find the 95% confidence limits for the difference of the mean wages of the population of the two companies. **(05 Marks)**

b. Five dice were thrown 96 times and the numbers 1,2 or 3 appearing on the face of the dice follows the frequency distribution as below.

Number of dice showing 1, 2 or 3	5	4	3	2	1	0
Frequency	7	19	35	24	8	3

Test the hypothesis that the data follows a binomial distribution $(\chi^2_{0.05} = 11.07 \text{ for 5 d.f.})$ **(05 Marks)**

c. A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand if he does not study one night, he is 60% sure not to study the next night. In the long run how often does he study? **(06 Marks)**

* * * * *