USN ☐☐☐☐☐☐☐☐☐☐

15MAT31

Third Semester B.E. Degree Examination, Aug./Sept. 2020
Engineering Mathematics – III

Time: 3 hrs.

Max. Marks: 80

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Obtain the Fourier series of $f(x) = x(2\pi - x)$ in $0 \le x \le 2\pi$ and hence deduce that :

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + - - -.$$

(08 Marks)

   b. Express y as a Fourier series upto the second harmonics given :

x	0	1	2	3	4	5
y	4	8	15	7	6	2

(08 Marks)

OR

2 a. Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2$. (06 Marks)
   b. Expand the function $f(x) = x\sin x$ as a Fourier series in the interval $-\pi \le x \le \pi$. (05 Marks)
   c. Expand $f(x) = 2x - 1$ as a cosine half range Fourier series in $0 \le x < 1$. (05 Marks)

Module-2

3 a. Find the Fourier transform of

$$f(x) = \begin{cases} 1-|x| & \text{for} \quad |x| \le 1 \\ 0 & \text{for} \quad |x| > 1 \end{cases}$$

And hence deduce that $\int_0^{10} \frac{\sin^2 t}{t^2} dt = \frac{\pi}{2}$

(06 Marks)

   b. Find the Fourier cosine transform of

$$f(x) = \begin{cases} x & 0 < x < 2 \\ 0 & \text{else where} \end{cases}.$$

(05 Marks)

   c. Find the z – transform of : i) $\cos n\theta$ ii) $\sin n\theta$. (05 Marks)

OR

4 a. Obtain the Fourier transform of $f(x) = x\,e^{-|x|}$. (06 Marks)
   b. If $u(z) = \dfrac{2z^2 + 3z}{(z+2)(z-4)}$, find the inverse z-transform. (05 Marks)
   c. Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using z – transforms. (05 Marks)

Module-3

5 a. Compute the co-efficient of correlation and equation of lines of regression for the data :

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

(06 Marks)

   b. Fit a best fitting parabola $y = ax^2 + bx + c$ for the following data :

x	1	2	3	4	5
y	10	12	13	16	19

(05 Marks)

   c. Use the Regula – Falsi method to find a real root of the equation $x^3 - 2x - 5 = 0$ correct to three decimal places. (05 Marks)

OR

6 a. Find the co-efficient of correlation for the following data :

x	10	14	18	22	26	30
y	18	12	24	6	30	36

(06 Marks)

   b. Fit a least square geometric curve $y = a e^{bx}$ for the following data :

x	0	2	4
y	8.12	10	31.82

(05 Marks)

   c. Use Newton – Raphson method to find a real root of the equation : $x \log_{10}^{x} = 1.2$ correct to four decimal places that is near to 2.5. (05 Marks)

Module-4

7 a. From the following table find the number of students who have obtained :
      i)   Less than 45 marks
      ii) Between 40 and 45 marks.

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Number of students	31	42	51	35	31

(06 Marks)

   b. Find the Legrange's interpolation polynomial for the following values $y(1) = 3$, $y(3) = 9$, $y(4) = 30$ and $y(6) = 132$. (05 Marks)

   c. Evaluate $\int_{0}^{1} \frac{dx}{1+x}$ taking seven ordinates by applying Simpson's $\frac{3}{8}^{th}$ rule. (05 Marks)

OR

8 a. Give $u_{20} = 24.37$, $u_{22} = 49.28$, $u_{29} = 162.86$ and $u_{32} = 240.5$ find u_{28} by Newton's divided difference formula. (06 Marks)

   b. Extrapolate for 25.4 given the data using Newton's backward formula :

x	19	20	21	22	23
y	91	100.25	110	120.25	131

(05 Marks)

   c. Evaluate : $\int_{0}^{1} \frac{x}{1+x^2} dx$ by Weddle's rule taking seven ordinates. (05 Marks)

Module-5

9 a. Verify Green's theorem for $\oint_C (xy + y^2)dx + x^2 dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. **(06 Marks)**

   b. Derive Euler's equation in the form $\dfrac{\partial f}{\partial y} - \dfrac{d}{dx}\left(\dfrac{\partial f}{\partial y^1}\right) = 0$. **(05 Marks)**

   c. If $\vec{F} = xyi + yzj + zxk$ evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the curve represented by $x = t$, $y = t^2$, $z = t^3$, $-1 \le t \le 1$. **(05 Marks)**

OR

10 a. Verify Green's theorem in the plane for $\int_C (x^2 + y^2)dx + 3x^2 y\, dy$ where C is the circle $x^2 + y^2 = 4$ traced in the positive sence. **(06 Marks)**

   b. Evaluate $\int_C (xy\,dx + xy^2 dy)$ by Stoke's theorem C is the square in the x-y plane with the vertices $(1, 0)$, $(-1, 0)$, $(0, 1)$ and $(0, 1)$. **(05 Marks)**

   c. Prove that the geodesics on a plane are straight lines. **(05 Marks)**

* * * * *