



CBCS SCHEME

18MAT21

Second Semester B.E. Degree Examination, Aug./Sept.2020 Advanced Calculus and Numerical Methods

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the angle between the surfaces $x^2 + y^2 - z^2 = 4$ and $z = x^2 + y^2 - 13$ at $(2, 1, 2)$. (06 Marks)
- b. If $\vec{F} = \nabla(xy^3z^2)$, find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ at $(1, -1, 1)$. (07 Marks)
- c. Find the value of the constant a such that the vector field $\vec{F} = (axy - z^3)\hat{i} + (a - 2)x^2\hat{j} + (1 - a)xz^2\hat{k}$ is irrotational and hence find a scalar function ϕ such that $\vec{F} = \nabla\phi$. (07 Marks)

OR

- 2 a. If $\vec{F} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ from $(0, 0, 0)$ to $(1, 1, 1)$ along the curve given by $x = t$, $y = t^2$ and $z = t^3$. (06 Marks)
- b. Use Green's theorem to find the area between the parabolas $x^2 = 4y$ and $y^2 = 4x$. (07 Marks)
- c. If $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and S is the rectangular parallelepiped bounded by $x = 0$, $y = 0$, $z = 0$ and $x = 2$, $y = 1$, $z = 3$. Find the flux across S . (07 Marks)

Module-2

- 3 a. Solve $(D^2 + 3D + 2)y = 4 \cos^2 x$. (06 Marks)
- b. Solve $(D^2 + 1)y = \sec x \tan x$, by the method of variation of parameter. (07 Marks)
- c. Solve $x^2 y'' + xy' + 9 = 3x^2 + \sin(3 \log x)$. (07 Marks)

OR

- 4 a. Solve $y'' + 2y' + y = 2x + x^2$. (06 Marks)
- b. Solve $(2x + 1)^2 y'' - 6(2x + 1)y' + 16y = 8(2x + 1)^2$. (07 Marks)
- c. The current i and the charge q in a series circuit containing an inductance L , capacitance C , emf E satisfy the differential equation: $L \frac{di}{dt} + \frac{q}{C} = E$; $i = \frac{dq}{dt}$. Express q and i in terms of t , given that L, C, E are constants and the value of i, q are both zero initially. (07 Marks)

Module-3

- 5 a. Form the partial differential equation by eliminating the arbitrary function from $\phi(xy + z^2, x + y + z) = 0$. (06 Marks)
- b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2 \sin y$ when $x = 0$ and $z = 0$ if $y = (2n + 1)\frac{\pi}{2}$. (07 Marks)
- c. Derive one dimensional wave equation in the standard form $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. (07 Marks)

OR

- 6 a. Form the partial differential equation by eliminating the arbitrary function form $f\left(\frac{xy}{z}, z\right) = 0$. (06 Marks)
- b. Solve $\frac{\partial^2 z}{\partial y^2} = z$, given that when $y = 0$, $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$. (07 Marks)
- c. Find all possible solutions of one dimensional heat equation $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ using the method of separation of variables. (07 Marks)

Module-4

- 7 a. Test for convergence of the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^n$, ($x > 0$). (06 Marks)
- b. Solve the Bessel's differential equation leading to $J_n(x)$. (07 Marks)
- c. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomials. (07 Marks)

OR

- 8 a. Test for convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$. (06 Marks)
- b. If α and β are two distinct roots of $J_n(x) = 0$. Prove that $\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = 0$. If $\alpha \neq \beta$. (07 Marks)
- c. Express $f(x) = x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomials. (07 Marks)

Module-5

- 9 a. Find the real root of the equation : $x^3 - 2x - 5 = 0$ using Regula Falsi method, correct to three decimal places. (06 Marks)
- b. Use Lagrange's formula, find the interpolating polynomial that approximates the function described by the following data :

x	0	1	2	5
f(x)	2	3	12	147

- c. Evaluate $\int_0^1 \frac{x dx}{1+x^2}$ by Weddle's rule, taking seven ordinates and hence find $\log e^2$.

OR

- 10 a. Find the real root of the equation $xe^x - 2 = 0$ using Newton - Raphson method correct to three decimal places.
- b. Use Newton's divided difference formula to find $f(4)$ given the data :

x	0	2	3	6
f(x)	-4	2	14	158

- c. Use Simpson's $\frac{3}{8}$ rule to evaluate $\int_1^4 e^{\frac{1}{x}} dx$.
