CBCS SCHEME

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16/17SCS/SCN/SCE/SSE/LNI/SFC/SIT14

First Semester M.Tech. Degree Examination, Aug./Sept.2020
Probability Statistics and Queuing Theory

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. In the experiment of tossing of two coins the following events are defined.
A = The first coin shown H. ≡ Head
B = The second coin shows T. ≡ Tail.
Find the probability that,
i) A occurs ii) A occurs given that B has already occurred.
What is the effect of occurrence of B on the probability of A? **(05 Marks)**

b. A student knew correct answers to only 60% of the questions in a test each with 5 alternate answers. He guessed while answering the rest of the questions. What is the probability that he know the correct answer to a question given that he answered it correctly? Use total probability and Baye's rule. **(06 Marks)**

c. The random variables X and Y have joint probability density function given by

$$f(X,Y)=\begin{cases}4xy\,e^{-(x^2+y^2)}, & x\geq 0\ y\geq 0\\ 0 & \text{otherwise}\end{cases}$$

Find the marginal distributions of X and Y. Check whether X and Y are independent. **(05 Marks)**

OR

2 a. Two-thirds of students in a class are boys and the rest are girls. It is known that the probability of a girl getting first class is 0.25 and that of a boy is 0.28. Find the probability that a student chosen at random will get first class. Use total probability. **(05 Marks)**

b. A lot of 10 items contains 3 defectives from which a sample of 4 items is drawn without replacement. If X is the discrete random variable being the number of defective items in a sample, find: i) the probability distribution of X ii) $p(X<1)$ iii) $p(0<X<2)$. **(06 Marks)**

c. A continuous random variable X has the PDF $f(X)=\begin{cases}2X & \text{if } 0\leq X\leq 1\\ 0 & \text{otherwise}\end{cases}$

i) Find Cumulative Distribution Function (CDF).
ii) Sketch PDF, CDF
iii) Find $P(1/4<x\leq 3/4)$, $P(x<1/2)$ **(05 Marks)**

Module-2

3 a. A coin is biased so that a head is twice as likely to appear as a tail. If the coin is tossed 6 times find the probabilities of getting
i) exactly 2 heads
ii) at least 3 heads
iii) at the most 4 heads. **(08 Marks)**

b. If X is a uniform random variable over the interval (a, b) then prove that the following properties hold:

$$\text{Mean}(X)=\frac{a+b}{2},\quad \text{Var}(X)=\frac{(b-a)^2}{12},\quad \text{Median}(X)=\frac{a+b}{2}$$ also find Mode(X). **(08 Marks)**

OR

4 a. The savings bank account of a customer showed an average balance of Rs.150 and standard deviation of Rs.50. Assuming that the account is normally distributed, find what percentage of account is i) over Rs.200 ii) between Rs.120 and Rs.170 iii) less than Rs.75 **(08 Marks)**

b. If X has geometric distribution with parameter 'p' find,
   i) p (X is even)
   ii) p (X is odd) **(08 Marks)**

Module-3

5 a. Define Random Process: Also state four types of stochastic process. **(05 Marks)**

b. A stationary random process $X = \{X(t)\}$ with mean 3 has auto-correlation function, $R(\tau) = 16 + 9\, e^{-|\tau|}$. Find the standard deviation of the process. **(06 Marks)**

c. If $X(t) = c$ where $c \equiv$ constant is a random process, examine whether $X(t)$ is mean ergodic. **(05 Marks)**

OR

6 a. Define average (mean) value of random variable $X(t)$. If $X(t) = A\cos(wt + \theta)$ when A and θ independent uniform variables over $(-k\ k)$ and $(-\pi\ \pi)$ respectively find mean $X(t)$. **(06 Marks)**

b. For the example $X(t) = A\cos(wt + \theta)$ mentioned in the question 6(a) find the autocorrelation function of $X(t)$. **(05 Marks)**

c. Given the transition matrix of a Markov chain, $A = \begin{matrix} & X & Y \\ X & .5 & .5 \\ Y & 0.1 & .9 \end{matrix}$ representing whether model of a day in two states X(rainy) or Y(sunny), find the whether possibilities on day 1, day 2, day 3 considering day '0' [day zero] is a "rainy" day. **(05 Marks)**

Module-4

7 a. Define,
   i) Statistical Hypothesis
   ii) Type I, Type II errors
   iii) Null hypothesis. **(05 Marks)**

b. A certain stimulus administered to each of 12 patients resulted the following change in the blood pressure 5, 2, 8, -1, 3, 0, -2, 1, 5, 0, 4, 6 can it be concluded of 5% level of significance that stimulus is in general accompanied by an increase in blood pressure. Use t-distribution given that $t_{0.05} = 2.2$ at 11 degree of freedom. **(06 Marks)**

c. In experiments on pea breeding the following frequencies of seeds were obtained:

Colour/Type	Wrinkled and yellow	Round and Green	Wrinkled and green	Round and yellow
Frequency	101	108	32	315

Theory predicts that frequencies should be in proportion to 3:3:1:9. Examine the correspondence between theory and experiment. Use χ^2 test. Given that $\chi^2_{0.05} = 7.815$ for '3' degrees of freedom. **(05 Marks)**

OR

8 a. Define level of significance. Draw a neat diagram to show the critical region to show 5% level of significance write down 95% and 99% confidence intervals for $x \sim N(\mu, 6)$. **(05 Marks)**

b. Measurements on the length of a copper wire taken M and experiments A and B are

A′s measurement (x_i) (mm)	12.29	12.25	11.86	12.13	12.44	12.78	12.77	11.90	12.47
B′s measurement y_i (mm)	12.39	12.46	12.34	12.22	11.98	12.46	12.23	12.06	

Test whether B's measurements are more accurate than A's (both reading are unbiased) use F-test. Given: for (8, 7) degrees of freedom $F_{0.05} = 3.73$ and $F_{0.01} = 6.84$ **(05 Marks)**

c. The following gives the data regarding colour of eges of Fathers and sons. Test the independence of eye colour of father and son. Use χ^2 test. Give χ^2 at 5% level for 1df = 3.841.

Eye colour of son

Eye colour of father		Light	Not Light
	Light	471	51
	Not light	148	230

(06 Marks)

Module-5

9 a. What is queuing theory? Write maximum two lines. What is the fundamental goal of queuing theory? Write in ONE sentence. Draw a simple queuing model. **(05 Marks)**

b. A super market has a single cashier. During peak hours, customers arrive at the rate of 20 per hour. The average number of customers that can be processed by the cashier is 24 per hour. Calculate:

i) Probability that cashier is idle

ii) The average number of customers in the queue

iii) The average time a customer spends in the queue for service. **(06 Marks)**

c. A telephone exchange has two long-distance operators. Long telephone calls arrive according to poisson law, at an average rate of 15 per hour. The length of service on these calls follows exponential law with mean length 2 minutes. Find the probability that no customer is in the queue. **(05 Marks)**

OR

10 a. i) Mention the four characteristics of a queuing system.

ii) Mention four queue disciplines. **(08 Marks)**

b. Explain pure birth process as a continuous time Markov-process. Represent it as family of staircase functions taking X(t) along y-axis and time t along X-axis. Also give the mathematical definition. **(08 Marks)**

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