AI-14
AF23 ①

CBCS SCHEME

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16/17SCS/SCN/SCE/SSE/LNI/SFC/SIT14

First Semester M.Tech. Degree Examination, Dec.2019/Jan.2020
Probability, Statistics and Queuing Theory

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. If A is any event, Prove $P(\overline{A}) = 1 - P(A)$. **(04 Marks)**

b. State and prove Baye's theorem. **(06 Marks)**

c. A given lot of 1C chips contain 2% defective chips. Each chip is tested before delivery. The tester itself is not totally reliable. Probability of tester says the chip is good when it really good is 0.95 and chip is defective when it actually defective is 0.94. If a tested device is indicated to be defective, what is the probability that it is actually defective? **(06 Marks)**

OR

2 a. A continuous random variable X has a probability density function $f(x) = k(1+x),\ 2 \le x \le 5$, Find $P(x \le 4)$. **(05 Marks)**

b. A random variable X has the following distribution,

X	−2	−1	0	1	2	3
P(X)	0.1	K	0.2	2K	0.3	3K

i) Find K ii) Evaluate $P(X < 2)$. **(05 Marks)**

c. Two balls are drawn at random without replacement from a box containing 2 green, 2 blue, and 1 red ball. If X denotes the number of blue balls drawn and Y denotes the number of red balls drawn, find the joint probability distribution of (X,Y). **(06 Marks)**

Module-2

3 a. Write the binomial, Poisson and geometric probability distribution functions along with their mean and variance. **(05 Marks)**

b. If X is a binomial distribution RV with $E(X) = 2$ and $Var(X) = 4/3$, find $P(X = 5)$. **(05 Marks)**

c. The number of monthly breakdowns of the computer is a RV having a Poisson distribution with mean equal 1.8. Find the probability that this computer will function for a month
i) Without a break down
ii) With only one breakdown
iii) With at least one breakdown. **(06 Marks)**

OR

4 a. Write the uniform, exponential and normal probability distribution functions with their mean and variance. **(05 Marks)**

b. The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = 1/3$. What is the probability that the repair time exceeds 3 hours? **(05 Marks)**

c. Random variable X has a uniform distribution with expected value of 10 and standard deviation of $\sqrt{3}$. Find :
i) $P(X < 9)$
ii) $P(10 < X < 15)$
iii) $P(X > 12)$. **(06 Marks)**

Module-3

5 a. State the four types of random processes. What is the difference between deterministic and non-deterministic processes? **(04 Marks)**

b. Show that the random process $x(t) = A\cos(w_0 t + \theta)$ is wide sense stationary, if and w_0 are constants and θ is a uniformly distributed RV is $(0, 2\pi)$. **(06 Marks)**

c. Consider a Markov chain with three possible states 1, 2 and 3 and the following transition probabilities P.

$$\begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

i) Find $P(X_4 = 3 \mid X_3 = 2)$
ii) Find $P(X_3 = 1 \mid X_2 = 1)$
iii) If we known $P(X_0 = 1) = 13$, Find $P(X_0 = 1, X_1 = 2)$ **(06 Marks)**

OR

6 a. Define Poisson process. What is the difference between homogeneous and non-homogeneous process? **(04 Marks)**

b. Show that the inter arrival time of a Poisson process with parameter X has an exponential distribution with mean $1/\lambda$. **(06 Marks)**

c. Queries presented in a computer database are following a Poisson process of rate $\lambda = 6$ queries per minute. An experiment consists of monitoring the database for 'm' minutes and recording N(m) the number of queries presented. What is the probability of
i) N queries one minute interval
ii) Exactly 6 queries arriving in a one minute interval
iii) Less than 3 quires arriving in a half minute interval. **(06 Marks)**

Module-4

7 a. Write a procedure for testing of hypothesis. **(05 Marks)**

b. What is the test metric for tests of significance of the difference between
i) Sample proportion and population proportion
ii) Two sample proportions
iii) Sample mean and population mean. **(06 Marks)**

c. A coin was flipped 60 times and came up heads 38 times. At the 0.10 level of significance, is the coin biased toward heads? Show your decision rule and calculations ($z_{0.10} = 1.28$). **(05 Marks)**

OR

8 a. The mean lifetime of a sample of 25 bulbs is found to be 1550 hours with SD of 120h. The company manufacturing the bulbs claims the average life of the bulbs is 1600h. Is the claim acceptable at 5% LOS? ($t_{0.05} = 1.71$) **(05 Marks)**

b. The following data give the number of air-craft accidents that occurred during the various days of a week.

Day	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents	15	19	13	12	16	15

Test whether the accidents are uniformly distributed over the week ($\chi^2_{0.05} = 11.07$). **(06 Marks)**

c. A sample of size 13 gave an estimated population variance of 3.0, while another sample of size 15 gave an estimate of 2.5, could both samples be from populations with the same variance? ($F_{0.05}$ (12, 14) = 2.53). **(05 Marks)**

Module-5

9 a. Explain symbolic representation a/b/c:d/e of the queuing model. **(05 Marks)**

b. State Little's Law regarding
   i) The number of customers in the system
   ii) Number of customers in the queue
   iii) Average waiting time in the system
   iv) Average waiting time in the queue. **(05 Marks)**

c. Suppose the customers arrive at Poison rate of one per every 12 minutes and that the service time is exponential at a rate of one service per 8 minutes. What is
   i) The average number of customers in the system
   ii) The average time a customer spends in the system. **(06 Marks)**

OR

10 a. In a public telephone booth having just one phone, the arrivals are considered to be Poisson with the average of 15 per hour. The length of a phone call is assumed to be distributed exponentially with mean 3 minutes. Find the
   i) Average number of customers waiting in the system
   ii) Average number of customers waiting in the queue
   iii) Probability that a person arriving at the booth will have to wait in the queue. **(06 Marks)**

b. A petrol pump station has 4 pumps. The service times follow the exponential distribution with a mean 6 min and cars arrive for service in a Poisson process at the rate of 30 cars per hour. For what percentage of time would a pump be idle on an average? **(06 Marks)**

c. Explain birth – death process. **(04 Marks)**

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