



Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Additional Mathematics – I

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find modulus and amplitude of $1 - \cos\theta + i \sin\theta$. (05 Marks)
- b. Express $\frac{3+4i}{3-4i}$ in $a + ib$ form. (05 Marks)
- c. Find the value of ' λ ' so that the points $A(-1, 4, -3)$, $B(3, 2, -5)$, $C(-3, 8, -5)$ and $D(-3, \lambda, 1)$, may lie on one plane. (06 Marks)

OR

- 2 a. Find the angle between the vectors $\vec{a} = 5\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$. (05 Marks)
- b. Prove that $\left[\begin{matrix} \vec{a} \times \vec{b} \\ \vec{b} \times \vec{c} \\ \vec{c} \times \vec{a} \end{matrix} \right] = \left[\begin{matrix} \vec{a} \\ \vec{b} \\ \vec{c} \end{matrix} \right]^2$. (05 Marks)
- c. Find the real part of $\frac{1}{1 + \cos\theta + i \sin\theta}$. (06 Marks)

Module-2

- 3 a. Obtain the n^{th} derivative of $\sin(ax + b)$. (05 Marks)
- b. Find the pedal equation of $r^n = a^n \cos n\theta$. (05 Marks)
- c. If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$, show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 4$. (06 Marks)

OR

- 4 a. If $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$. (05 Marks)
- b. If $u = f(x - y, y - z, z - x)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (05 Marks)
- c. If $y = a \cos(\log x) + b \sin(\log x)$, show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2 + 1)y_n = 0$ (06 Marks)

Module-3

- 5 a. Evaluate $\int_0^\pi x \sin^8 x dx$. (05 Marks)
- b. Evaluate $\int_0^1 x^2 (1-x^2)^{3/2} dx$. (05 Marks)
- c. Evaluate $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx$. (06 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and /or equations written eg, $42+8 = 50$, will be treated as malpractice.

OR

- 6 a. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy dy dx$. (05 Marks)
- b. Evaluate $\int_0^1 \int_0^1 \int_0^1 (x+y+z) dx dy dz$. (05 Marks)
- c. Evaluate $\int_0^{\infty} \frac{x^4}{(1+x^2)^4} dx$. (06 Marks)

Module-4

- 7 a. If $\vec{r} = (t^2 + 1)\hat{i} + (4t - 3)\hat{j} + (2t^2 - 6t)\hat{k}$, find the angle between the tangents at $t = 1$ and $t = 2$. (05 Marks)
- b. If $\vec{r} = e^{-t}\hat{i} + 2\cos 3t\hat{j} + 2\sin 3t\hat{k}$, find the velocity and acceleration at any time t , and also their magnitudes at $t = 0$. (05 Marks)
- c. Show that $\vec{F} = (y+z)\hat{i} + (z+x)\hat{j} + (x+y)\hat{k}$ is irrotational. Also find a scalar function ' ϕ ' such that $\vec{F} = \nabla\phi$. (06 Marks)

OR

- 8 a. Find the unit normal vector to the surface $x^2y + 2xz = 4$ at $(2, -2, 3)$. (05 Marks)
- b. If $\vec{F} = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$ find $\nabla \cdot \vec{F}$ and $\nabla \times \vec{F}$ at $(1, -1, 1)$. (05 Marks)
- c. If $\frac{d\vec{a}}{dt} = \vec{w} \times \vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{w} \times \vec{b}$, then show that $\frac{d}{dt}(\vec{a} \times \vec{b}) = \vec{w} \times (\vec{a} \times \vec{b})$ (06 Marks)

Module-5

- 9 a. Solve $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$. (05 Marks)
- b. Solve $(y^3 - 3x^2y)dx + (3xy^2 - x^3)dy = 0$. (05 Marks)
- c. Solve $\frac{dy}{dx} + \frac{y}{x} = xy^2$. (06 Marks)

OR

- 10 a. Solve $\frac{dy}{dx} + y \cot x = \cos x$. (05 Marks)
- b. Solve $x^2 y dx - (x^3 + y^3) dy = 0$ (05 Marks)
- c. Solve $y(x+y)dx + (x+2y-1)dy = 0$ (06 Marks)
