



## Third Semester B.E. Degree Examination, Dec.2019/Jan.2020 Additional Mathematics – I

Time: 3 hrs.

Max. Marks: 100

**Note:** Answer any FIVE full questions, choosing ONE full question from each module.

### Module-1

- 1 a. Express the following complex number in the form of  $x + iy$ :  $\frac{(1+i)(1+3i)}{1+5i}$ . (06 Marks)
- b. Prove that  $\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^4 = \cos 8\theta + i\sin 8\theta$ . (07 Marks)
- c. If  $\vec{a} = (3, -1, 4)$ ,  $\vec{b} = (1, 2, 3)$  and  $\vec{c} = (4, 2, -1)$ , find  $\vec{a} \times (\vec{b} \times \vec{c})$ . (07 Marks)

**OR**

- 2 a. Find the angle between the vectors,  $\vec{a} = 5\hat{i} - \hat{j} + \hat{k}$  and  $\vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ . (06 Marks)
- b. Prove that  $\left[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}\right] = \left[\vec{a}, \vec{b}, \vec{c}\right]^2$  (07 Marks)
- c. Find the fourth roots of  $-1 + i\sqrt{3}$  and represent them on the argand diagram. (07 Marks)

### Module-2

- 3 a. Obtain the Maclaurin's expansion of  $\log_e(1+x)$ . (06 Marks)
- b. If  $u = \sin^{-1}\left[\frac{x^3 + y^3}{x+y}\right]$ , prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \tan u$ . (07 Marks)
- c. If  $u = x(1-y)$ ,  $v = xy$ , find  $\frac{\partial(u,v)}{\partial(x,y)}$ . (07 Marks)

**OR**

- 4 a. Obtain the Maclaurin's series expansion of the function  $\log_e \sec x$ . (06 Marks)
- b. If  $u = x^2 - 2y$ ;  $v = x + y$  find  $\frac{\partial(u,v)}{\partial(x,y)}$ . (07 Marks)
- c. If  $u = f(x-y, y-z, z-x)$ , prove that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ . (07 Marks)

### Module-3

- 5 a. Find the velocity and acceleration of a particle moves along the curve,  $\vec{r} = e^{-2t}\hat{i} + 2\cos 5t\hat{j} + 5\sin 2t\hat{k}$  at any time  $t$ . (06 Marks)
- b. Find  $\text{div } \vec{F}$  and  $\text{curl } \vec{F}$ , where  $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ . (07 Marks)
- c. Show that  $\vec{F} = (2xy + z^2)\hat{i} + (x^2 + 2yz)\hat{j} + (y^2 + 2xz)\hat{k}$  is conservative force field and find the scalar potential. (07 Marks)

OR

- 6 a. Show that the vector field,  $\vec{F} = (3x + 3y + 4z)\hat{i} + (x - 2y + 3z)\hat{j} + (3x + 2y - z)\hat{k}$  is solenoidal. (06 Marks)
- b. Find the directional derivative of  $\phi = \frac{xz}{x^2 + y^2}$  at  $(1, -1, 1)$  in the direction of  $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$ . (07 Marks)
- c. Find the constant 'a' such that the vector field  $\vec{F} = 2xy^2z^2\hat{i} + 2x^2yz^2\hat{j} + ax^2y^2z\hat{k}$  is irrotational. (07 Marks)

Module-4

- 7 a. Find the reduction formula for  $\int_0^{\frac{\pi}{2}} \sin^n x dx$ . (06 Marks)
- b. Evaluate  $\int_0^1 \int_0^3 x^3 y^3 dx dy$ . (07 Marks)
- c. Evaluate  $\int_0^3 \int_0^2 \int_0^1 (x + y + z) dz dx dy$ . (07 Marks)

OR

- 8 a. Evaluate :  $\int_0^{\frac{\pi}{6}} \sin^6(3x) dx$ . (06 Marks)
- b. Evaluate :  $\int_0^1 \int_x^{\sqrt{x}} xy dy dx$ . (07 Marks)
- c. Evaluate :  $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz dz dy dx$ . (07 Marks)

Module-5

- 9 a. Solve :  $\frac{dy}{dx} + y \cot x = \sin x$ . (06 Marks)
- b. Solve :  $(2x^3 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy = 0$ . (07 Marks)
- c. Solve :  $3x(x + y^2)dy + (x^3 - 3xy - 2y^3)dx = 0$ . (07 Marks)
- 10 a. Solve :  $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$ . (06 Marks)
- b. Solve :  $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ . (07 Marks)
- c. Solve :  $[1 + (x + y) \tan y] \frac{dy}{dx} + 1 = 0$ . (07 Marks)

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