CBCS SCHEME

USN

17MATDIP31

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the modulus and amplitude of $\dfrac{3+i}{2+i}$ **(07 Marks)**

b. If $x = \cos\theta + i\sin\theta$, then show that $\dfrac{x^{2n}-1}{x^{2n}+1} = i\tan n\theta$. **(07 Marks)**

c. Simplify $\dfrac{(\cos 3\theta + i\sin 3\theta)^4(\cos 4\theta + i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3(\cos 5\theta + i\sin 5\theta)^{-4}}$ **(06 Marks)**

OR

2 a. Find the sine of the angle between $\vec{A} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = 6\hat{i} - 3\hat{j} + 2\hat{k}$. **(07 Marks)**

b. Find the value of λ, so that the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = \hat{i} + \lambda\hat{k}$ are coplanar. **(07 Marks)**

c. Prove that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$. **(06 Marks)**

Module-2

3 a. Find the n^{th} derivative of $e^{ax}\cos(bx + c)$. **(07 Marks)**

b. If $y = a\cos(\log x) + b\sin(\log x)$ prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(07 Marks)**

c. If $u = \sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$. **(06 Marks)**

OR

4 a. Find the pedal equation of $r^n = a^n \cos n\theta$. **(07 Marks)**

b. Expand $\log_e(1 + x)$ in ascending powers of x as for as the term containing x^4. **(07 Marks)**

c. If $x = r\cos\theta$, $y = r\sin\theta$, find $\dfrac{\partial(x, y)}{\partial(r, \theta)}$ **(06 Marks)**

Module-3

5 a. Evaluate $\displaystyle\int_0^1\int_{y^2}^{y}(1 + xy^2)\,dx\,dy$ **(07 Marks)**

b. Evaluate $\displaystyle\int_0^{2\pi}\sin^4 x\cos^6 x\,dx$ **(07 Marks)**

c. Evaluate $\displaystyle\int_0^2 \frac{x^4}{\sqrt{4-x^2}}\,dx$ **(06 Marks)**

OR

6 a. Evaluate $\int_1^2\int_3^4 (xy+e^y)\,dydx$ **(07 Marks)**

b. Evaluate $\int_0^\pi x\sin^8 x\,dx$ **(07 Marks)**

c. Evaluate $\int_1^2\int_0^1\int_{-1}^1 (x^2+y^2+z^2)\,dxdydz$ **(06 Marks)**

Module-4

7 a. If particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$ where t is the time. Find the velocity and acceleration at $t = 1$. **(07 Marks)**

b. Find the angle between the tangents to the curve $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$ at the point $t = \pm 1$. **(07 Marks)**

c. If $\vec{F} = (3x^2y - z)\hat{i} + (xz^3 + y^4)\hat{j} - 2x^3z^2\hat{k}$ find grad(div $\vec{F}$) at (2, –1, 0). **(06 Marks)**

OR

8 a. Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at (2, –1, 2) along $2\hat{i} - 3\hat{j} + 6\hat{k}$ **(07 Marks)**

b. Find the unit normal to the surface $x^2y + 2xz = 4$ at (2, –2, 3). **(07 Marks)**

c. Show that $\vec{F} = (2xy^2 + yz)\hat{i} + (2x^2y + xz + 2yz^2)\hat{j} + (2y^2z + xy)\hat{k}$ is irrotational. **(06 Marks)**

Module-5

9 a. Solve $\frac{dy}{dx} = \sin(x+y)$ **(07 Marks)**

b. Solve $\frac{dy}{dx} + y\cot x = \cos x$ **(07 Marks)**

c. Solve $(x - y + 1)dy - (x + y - 1)dx = 0$ **(06 Marks)**

OR

10 a. Solve $(1 + e^{x/4})dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$. **(07 Marks)**

b. Solve $(x^3\cos^2 y - x\sin 2y)\,dx = dy$. **(07 Marks)**

c. Solve $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$ **(06 Marks)**

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