USN

10MAT41

Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Solve $\frac{dy}{dx} = x^2y - 1$ with $y(0) = 1$, using Taylor's series method and find y(0.1) by considering upto fourth degree term. **(07 Marks)**

b. By using the Runge – Kutta method of order 4, solve the equation $\frac{dy}{dx} = 3x + \frac{y}{2}$ with y(0) = 1 at the point x = 0.1. Taking step length h = 0.1. **(07 Marks)**

c. By using Milne's method, solve the different equation : $\frac{dy}{dx} = \frac{2y}{x}$ $x \neq 0$ at the point x = 2 given that y(1) = 2, y(1.25) = 3.13, y(1.5) = 4.5 and y(1.75) = 6.13. Apply corrector formula twice. **(06 Marks)**

2 a. By Pieard's method, find the successive approximate solutions, upto 2nd order of the system of differential equations $\frac{dy}{dx} = x + z$, $\frac{dz}{dx} = x - y^2$ under the initial conditions y(0) = 2, z(0) = 1. Deduce the solutions at the point x = 0.1. **(07 Marks)**

b. By using the Picard's method, find the second order approximate solutions at x = 1.1 and 1.2 of the differential equation : $\frac{d^2y}{dx^2} + y^2\frac{dy}{dx} - x^3 = 0$, with y(1) = y′(1) = 1. **(07 Marks)**

c. Given $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1$, $y(0) = 1$, $y'(0) = 0$. Evaluate y(0.1) using Runge Kutta method of order 4. **(06 Marks)**

3 a. Derive Cauchy-Riemann equation in Cartesian form. **(07 Marks)**

b. Show that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic and find its harmonic conjugate. Also find the corresponding analytic function f(z). **(07 Marks)**

c. If f(z) is analytic function, show that $\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right]|f(z)|^2 = 4|f'(z)|^2$. **(06 Marks)**

4 a. Find the Bilinear transformation that maps 0, –i, –1 of z-plane onto the points i, 1, 0 of w-plane respectively. **(07 Marks)**

b. State and prove Cauchy's theorem. **(07 Marks)**

c. Evaluate : $\int_C \frac{c^{2z}}{(z+1)(z+2)} dz$, where C is the circle | z | = 3. **(06 Marks)**

PART – B

5 a. Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. (07 Marks)

b. Obtain the series solution of Legendre's differential equation. (07 Marks)

c. Expression $f(x) = x^3 + 2x^2 - 4x + 5$ in terms of Legendre polynomial. (06 Marks)

6 a. A problem is given to 3 students A, B, C whose chances of solving it are $\frac{2}{3}, \frac{3}{4}$ and $\frac{1}{4}$ respectively. Find the probability that problem is solved. (07 Marks)

b. State and prove Baye's theorem. (07 Marks)

c. Three students A, B, C write an entrance examination. Their chances of passing are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability that :
i) atleast one of them passes
ii) all of them pass. (06 Marks)

7 a. The probability distribution of a finite random variable X is given by the following table :

X	–2	–1	0	1	2	3
$P(X_i)$	0.1	K	0.2	2K	0.3	K

Find the value of K, mean and variance. (07 Marks)

b. The probability that a person aged 60 years will live upto 70 is 0.65. What is the probability that out of 10 persons aged 60 aleast 7 of them will live upto 70? (07 Marks)

c. Find the constant k such that $f(x) = \begin{cases} kx^2 & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$ is a p.d.f.

Also compute : i) $p(1 < x < 2)$ ii) $p(x \le 1)$ iii) $p(x > 1)$. (06 Marks)

8 a. A random sample of 400 items is found to have a mean of 82 and the standard deviation of 18. Find 95% confidence limits for the mean of the population from which the sample is drawn. (07 Marks)

b. A machine is expected to produce nails of length 3 inches. A random sample of 25 nails gave an average length of 3.1 inch with standard deviation 0.3. Can it be said that the machine is producing nails per specification? ($t_{0.05}$ for 24 d.f. is 2.064). (07 Marks)

c. A die is thrown 264 times and the number appearing on the face(x) follows the following frequency distribution :

x	1	2	3	4	5	6
f	40	32	28	58	54	60

Calculate the value of χ^2. (06 Marks)

* * * * *