CBCS SCHEME

USN | | | | | | | | | | |

15MAT41

Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 80

Note: 1. ***Answer FIVE full questions, choosing ONE full question from each module.***
2. ***Use of statistical table can be provided.***

Module-1

1 a. Using Taylor's series method find, y(0.1) given that $\frac{dy}{dx} = x - y^2$, y(0) = 1 by considering upto third degree terms. **(05 Marks)**

b. Apply Runge Kutta method of fourth order to find an approximate value of y when x = 0.5 given that $\frac{dy}{dx} = \frac{1}{x+y}$ with $y(0.4) = 1$. Take h = 0.1. **(05 Marks)**

c. Evaluate y(0.4) by Milne's Predictor-Corrector method given that $\frac{dy}{dx} = \frac{y^2(1+x^2)}{2}$ and y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21. Apply the corrector formula twice. **(06 Marks)**

OR

2 a. Solve by Euler's modified method $\frac{dy}{dx} = \log_e(x+y)$; y(0) = 2 to find y(0.2) with h = 0.2. Carryout two modifications. **(05 Marks)**

b. Using Runge-Kutta method of fourth order find y(0.2) to four decimal places given that $\frac{dy}{dx} = 3x + \frac{y}{2}$; $y(0) = 1$. Take h = 0.2. **(05 Marks)**

c. Given $\frac{dy}{dx} = x^2(1+y)$; y(1) =1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979. Evaluate y(1.4) to four decimal places using Adam's-Bashforth predictor corrector method. Apply the corrector formula twice. **(06 Marks)**

Module-2

3 a. Given $\frac{d^2y}{dx^2} = y + x\frac{dy}{dx}$ with y(0) = 1, $y'(0) = 0$. Evaluate y(0.2) using Runge Kutta method of fourth order. Take h = 0.2. **(05 Marks)**

b. With usual notation prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. **(05 Marks)**

c. Express $f(x) = 2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomial. **(06 Marks)**

OR

4 a. Apply Milnes predictor corrector method to compute y(0.4) given that $\frac{d^2y}{dx^2} = 6y - 3x\frac{dy}{dx}$ and the following values: **(05 Marks)**

x	0	0.1	0.2	0.3
y	1	1.03995	1.138036	1.29865
y′	0.1	0.6955	1.258	1.873

b. State Rodrigue's formula for Legendre polynomials and obtain the expression for $P_4(x)$ from it. **(05 Marks)**

c. If α and β are the two roots of the equation $J_n(x) = 0$ then prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$ if $\alpha \neq \beta$. **(06 Marks)**

Module-3

5 a. Derive Cauchy-Riemann equation in Cartesian form. **(05 Marks)**

b. Evaluate using Cauchy's residue theorem, $\int_C \frac{3z^2+z+1}{(z^2-1)(z+3)}dz$ where C is the circle $|z| = 2$. **(05 Marks)**

c. Find the bilinear transformation which maps the points $-1, i, 1$ onto the points 1, i, -1 respectively. **(06 Marks)**

OR

6 a. Find the analytic function, $f(z) = u + iv$ if $v = r^2\cos 2\theta - r\cos\theta + 2$. **(05 Marks)**

b. Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)}dz$ where C is the circle $|z| = 3$ using Cauchy integral formula. **(05 Marks)**

c. Discuss the transformation $\omega = e^z$. **(06 Marks)**

Module-4

7 a. Find the constant C such that the function,

$$f(x) = \begin{cases} Cx^2 & \text{for } 0 < x < 3 \\ 0 & \text{Otherwise} \end{cases}$$ is a probability density function.

Also compute $P(1<X<2)$, $P(X \leq 1)$, $P(X>1)$. **(05 Marks)**

b. Out of 800 families with five childrens each, how many families would you expect to have (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys (iv) at most 2 girls, assume equal probabilities for boys and girls. **(05 Marks)**

c. Given the following joint distribution of the random variables X and Y.

X \ Y	1	3	9
2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4	$\frac{1}{4}$	$\frac{1}{4}$	0
6	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

Find (i) E(X) (ii) E(Y) (iii) E(XY) (iv) COV(X, Y) (v) ρ(X, Y) **(06 Marks)**

OR

8 a. Obtain the mean and standard deviation of Poisson distribution. **(05 Marks)**

b. In a test on electric bulbs it was found that the life time of bulbs of a particular brand was distributed normally with an average life of 2000 hours and standard deviation of 60 hours. If a firm purchases 2500 bulbs find the number of bulbs that are likely to last for,
(i) More than 2100 hours (ii) Less than 1950 hours (iii) Between 1900 and 2100 hours.
Given that $\phi(1.67) = 0.4525$, $\phi(0.83) = 0.2967$ **(05 Marks)**

c. A fair coin is tossed thrice. The random variables X and Y are defined as follows:
X = 0 or 1 according as head or tail occurs on the first toss.
Y = number of heads
Determine (i) The distribution of X and Y (ii) Joint distribution of X and Y. **(06 Marks)**

Module-5

9 a. In a city A 20% of a random sample of 900 school boys had a certain slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant. **(05 Marks)**

b. The nine items of a sample have the following values : 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ from the assumed mean 47.5. Apply student's t – distribution at 5% level of significance ($t_{0.05} = 2.31$ for 8 d.f) **(05 Marks)**

c. Find the unique fixed probability vector of the regular stochastic matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$. **(06 Marks)**

OR

10 a. A sample of 100 tyres is taken from a lot. The mean life of tyres is found to be 40,650 kms with a standard deviation of 3260. Can it be considered as a true random sample from a population with mean life of 40,000 kms (use 0.05 level of significance) Establish 99% confidence limits within which the mean life of tyres is expected to lie, (given $Z_{0.05} = 1.96$, $Z_{0.01} = 2.58$) **(05 Marks)**

b. In the experiments of pea breeding the following frequencies of seeds were obtained.

Round and Yellow	Wrinkled and Yellow	Round and Green	Wrinkled and Green	Total
315	101	108	32	556

Theory predicts that the frequencies should be in proportions 9 : 3 : 3 : 1. Examine the correspondence between theory and experiment.
$\left(\chi^2_{0.05} = 7.815 \text{ for } 3\text{ d.f}\right)$ **(05 Marks)**

c. Three boys A, B, C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C. C is just as likely to throw the ball to B is to A. If C was the first person to throw the ball find the probabilities that after the three throws.
(i) A has the ball (ii) B has the ball (iii) C has the ball. **(06 Marks)**

* * * * *