CBCS SCHEME

USN

18MAT31

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find the Laplace transform of:

(i) $\left(\frac{4t+5}{e^{2t}}\right)^2$ (ii) $\left(\frac{\sin 2t}{\sqrt{t}}\right)^2$ (iii) $t\cos at$. **(10 Marks)**

b. The square wave function f(t) with period 2a defined by $f(t)=\begin{cases}1 & 0\le t<a\\ -1 & a\le t<2a\end{cases}$. Show that $\left(\frac{1}{s}\right)\tanh\left(\frac{as}{2}\right)$. **(05 Marks)**

c. Employ Laplace transform to solve $\frac{d^2y}{dt^2}-\frac{dy}{dt}=0$, $y(0)=y_1(0)=3$. **(05 Marks)**

OR

2 a. Find (i) $L^{-1}\left\{\frac{s^2-3s+4}{s^3}\right\}$ (ii) $\cot^{-1}\left(\frac{s}{2}\right)$ (iii) $L^{-1}\left\{\frac{s}{(s+2)(s+3)}\right\}$ **(10 Marks)**

b. Find the inverse Laplace transform of, $\frac{1}{s(s^2+1)}$ using convolution theorem. **(05 Marks)**

c. Express $f(t)=\begin{cases}2 & \text{if } 0<t<1\\ \frac{t^2}{2} & \text{if } 1<t<\frac{\pi}{2}\\ \cos t & t>\frac{\pi}{2}\end{cases}$ in terms of unit step function and hence find its Laplace transformation. **(05 Marks)**

Module-2

3 a. Obtain the Fourier series of $f(x)=\begin{cases}2 & -2<x<0\\ x & 0<x<2\end{cases}$. **(08 Marks)**

b. Find the half range cosine series of, $f(x)=(x+1)$ in the interval $0\le x\le 1$. **(06 Marks)**

c. Express $f(x)=x^2$ as a Fourier series of period 2π in the interval $0<x<2\pi$. **(06 Marks)**

OR

4 a. Compute the first two harmonics of the Fourier Series of f(x) given the following table :

x°	0	60°	120°	180°	240°	300°
y	7.9	7.2	3.6	0.5	0.9	6.8

(08 Marks)

b. Find the half range size series of e^x in the interval $0 \le x \le 1$. **(06 Marks)**

c. Obtain the Fourier series of $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4}$ valid in the interval $(-\pi \quad \pi)$ **(06 Marks)**

Module-3

5 a. Find the Infinite Fourier transform of $e^{-|x|}$. **(07 Marks)**

b. Find the Fourier cosine transform of $f(x) = e^{-2x} + 4e^{-3x}$. **(06 Marks)**

c. Solve $u_{n+2} - 3u_{n+1} + 2u_n = 3^n$, given $u_0 = u_1 = 0$. **(07 Marks)**

OR

6 a. If $f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$, find the infinite transform of f(x) and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(07 Marks)**

b. Obtain the Z-transform of $\cosh n\theta$ and $\sinh n\theta$. **(06 Marks)**

c. Find the inverse Z-transform of $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$ **(07 Marks)**

Module-4

7 a. Solve $\frac{dy}{dx} = e^x - y$, $y(0) = 2$ using Taylor's Series method upto 4^{th} degree terms and find the value of $y(1.1)$. **(07 Marks)**

b. Use Runge-Kutta method of fourth order to solve $\frac{dy}{dx} + y = 2x$ at x = 1.1 given $y(1) = 3$ (Take $h = 0.1$) **(06 Marks)**

c. Apply Milne's predictor-corrector formulae to compute y(0.4) given $\frac{dy}{dx} = 2e^x y$, with **(07 Marks)**

x	0	0.1	0.2	0.3
y	2.4	2.473	3.129	4.059

OR

8 a. Given $\frac{dy}{dx} = x + \sin y$; $y(0) = 1$. Compute y(0.4) with h = 0.2 using Euler's modified method. **(07 Marks)**

b. Apply Runge-Kutta fourth order method, to find y(0.1) with h = 0.1 given $\frac{dy}{dx} + y + xy^2 = 0$; y(0) = 1. **(06 Marks)**

c. Using Adams-Bashforth method, find y(4.4) given $5x\left(\frac{dy}{dx}\right) + y^2 = 2$ with

x	4	4.1	4.2	4.3
y	1	1.0049	1.0097	1.0143

(07 Marks)

Module-5

9 a. Solve by Runge Kutta method $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for x = 0.2 correct 4 decimal places, using initial conditions $y(0) = 1,\ y'(0) = 0,\ h = 0.2$. **(07 Marks)**

b. Derive Euler's equation in the standard form, $\frac{\partial f}{\partial y} - \frac{d}{dx}\left[\frac{\partial f}{\partial y'}\right] = 0$. **(06 Marks)**

c. Find the extramal of the functional, $\int_{x_1}^{x_2} y^2 + (y')^2 + 2ye^x dx$. **(07 Marks)**

OR

10 a. Apply Milne's predictor corrector method to compute $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ and the following table of initial values:

x	0	0.1	0.2	0.3
y	1	1.1103	1.2427	1.3990
y′	1	1.2103	1.4427	1.6990

(07 Marks)

b. Find the extramal for the functional, $\int_0^{\frac{\pi}{2}} \left[y^2 - y'^2 - 2y\sin x\right]dx$; y(0) = 0; $y\left(\frac{\pi}{2}\right) = 1$. **(06 Marks)**

c. Prove that geodesics of a plane surface are straight lines. **(07 Marks)**

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