15MAT21

Second Semester B.E. Degree Examination, Dec.2019/Jan.2020 **Engineering Mathematics - II**

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Solve
$$\frac{dy^2}{dx^2} - 4y = \cosh(2x - 1) + 3^x$$
 by inverse differential operators method. (06 Marks)

c. Solve
$$(D^2 + a^2)$$
 y = Sec (ax) by the method of variation of parameters. (05 Marks)

b. Solve
$$(D^3 + D^2 + 4D + 4)$$
 $y = x^2 - 4x - 6$ by inverse differential operator method. (05 Marks)

c. Solve
$$y'' - 2y' + 3y = x^2 - \cos x$$
 by the method of undetermined coefficients. (05 Marks)

3 a. Solve
$$x^3y''' + 3x^2y'' + xy' + 8y = 65 \cos(\log x)$$
 (06 Marks)

b. Solve
$$xy\left(\frac{dy}{dx}\right)^2 - (x^2 + y^2)\frac{dy}{dx} + xy = 0$$
 (05 Marks)

c. Solve the equation
$$(px - y)$$
 $(py + x) = 2p$ by reducing into Clairaut's form taking the substitution $X = x^2$, $Y = y^2$. (05 Marks)

4 a. Solve
$$(2x-1)^2y'' + (2x-1)y' - 2y = 8x^2 - 2x + 3$$
 (06 Marks)

b. Solve
$$y = 2px + p^2y$$
 by solving for 'x'. (05 Marks)

c. Find the general and singular solution of equation
$$xp^2 - py + kp + a = 0$$
. (05 Marks)

Module-3

a. Obtain partial differential equation by eliminating arbitrary function.

Given
$$z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$$
. (06 Marks)

b. Solve
$$\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$$
, given that $u = 0$ when $t = 0$ and $\frac{\partial u}{\partial t} = 0$ at $x = 0$. (05 Marks)

c. Derive one dimensional wave equation
$$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$$
 (05 Marks)

a. Obtain partial differential equation of

$$f(x^2 + 2yz, y^2 + 2zx) = 0.$$
 (05 Marks)

b. Solve
$$\frac{\partial^2 z}{\partial x^2} = a^2 z$$
, given that when $x = 0$, $z = 0$ and $\frac{\partial z}{\partial x} = a \sin y$. (05 Marks)

c. Find the solution of one dimensional heat equation
$$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$$
. (05 Marks)

Module-4

7 a. Evaluate
$$\int_{0}^{1} \int_{0}^{1-x} \int_{0}^{1-x-y} \frac{dx \, dy \, dx}{(1+x+y+z)^3}$$
. (06 Marks)

b. Evaluate integral
$$\int_{0}^{1} \int_{0}^{x} xy \, dy \, dx$$
 by changing the order of integration. (05 Marks)

c. Obtain the relation between Beta and Gamma function in the form
$$\beta(m,n) = \frac{\overline{|m|n}}{\overline{|m+n|}}$$
 (05 Marks)

OR

8 a. Evaluate
$$\int_{0.0}^{\infty} e^{-(x^2+y^2)} dxdy$$
 by changing into polar co-ordinates. (06 Marks)

b. If A is the area of rectangular region bounded by the lines
$$x = 0$$
, $x = 1$, $y = 0$, $y = 2$ then evaluate $\int (x^2 + y^2) dA$.

c. Evaluate
$$\int_{0}^{\pi/2} \frac{d\theta}{\sqrt{\sin \theta}} \int_{0}^{\pi/2} \sqrt{\sin \theta} d\theta \text{ using Beta and Gamma functions.}$$
 (05 Marks)

Module-5

9 a. Find Laplace transition of i)
$$t^2e^{2t}$$
 ii) $\frac{e^{-at}-e^{-bt}}{t}$. (06 Marks)

b. If a periodic function of period 2a is defined by
$$f(t) = \begin{cases} t & \text{if } 0 \le t \le a \\ 2a - t & \text{if } a \le t \le 2a \end{cases}$$
 Then show that $L\{f(t)\} = \frac{1}{s^2} \tanh\left(\frac{as}{2}\right)$. (05 Marks)

c. Solve
$$y''(t) + 4y'(t) + 4y(t) = e^t$$
 with $y(0) = 0$ $y'(0) = 0$. Using Laplace transform. (05 Marks)

OR

10 a. Find
$$L^{-1} \left[\frac{7s}{(4s^2 + 4s + 9)} \right]$$
 (06 Marks)
b. Find $L^{-1} \left[\frac{s}{(s-1)(s^2 + 4)} \right]$ using convolution theorem. (05 Marks)