

# CBCS SCHEME

18MAT21

USN

## Second Semester B.E. Degree Examination, Dec.2019/Jan.2020 Advanced Calculus and Numerical Methods

Time: 3 hrs.

Max. Marks: 100

*Note: Answer any FIVE full questions, choosing ONE full question from each module.*

### Module-1

- 1 a. Find the directional derivative of  $\phi = 4xz^3 - 3x^2y^2z$  at  $(2, -1, 2)$  along  $2\bar{i} - 3\bar{j} + 6\bar{k}$ . (06 Marks)
- b. If  $\bar{f} = \nabla(x^3 + y^3 + z^3 - 3xyz)$  find  $\text{div } \bar{f}$  and  $\text{curl } \bar{f}$ . (07 Marks)
- c. Find the constants  $a$  and  $b$  such that  $\bar{F} = (axy + z^3)\bar{i} + (3x^3 - z)\bar{j} + (bxz^2 - y)\bar{k}$  is irrotational. Also find a scalar potential  $\phi$  if  $\bar{F} = \nabla\phi$ . (07 Marks)

OR

- 2 a. If  $\bar{F} = xy\bar{i} + yz\bar{j} + zx\bar{k}$  evaluate  $\int_C \bar{F} \cdot d\bar{r}$  where  $C$  is the curve represented by  $x = t, y = t^2, z = t^3, -1 \leq t \leq 1$ . (06 Marks)
- b. Using Stoke's theorem Evaluate  $\oint_C \bar{F} \cdot d\bar{r}$  if  $\bar{F} = (x^2 + y^2)\bar{i} - 2xy\bar{j}$  taken round the rectangle bounded by  $x = 0, x = a, y = 0, y = b$ . (07 Marks)
- c. Using divergence theorem, evaluate  $\iiint_S \bar{F} \cdot \bar{n} \, ds$  if  $\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$  taken around  $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$ . (07 Marks)

### Module-2

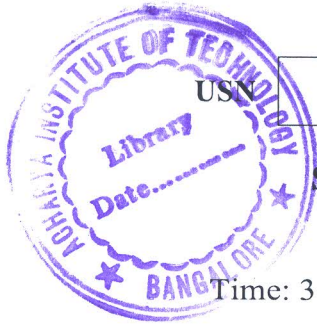
- 3 a. Solve  $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$  (06 Marks)
- b. Solve  $(D^2 + 4D + 3)y = e^{-x}$  (07 Marks)
- c. Using the method of variation of parameter solve  $y'' + 4y = \tan 2x$ . (07 Marks)

OR

- 4 a. Solve  $(D^3 - 1)y = 3 \cos 2x$  (06 Marks)
- b. Solve  $x^2y'' - 5xy' + 8y = 2 \log x$  (07 Marks)
- c. The differential equation of a simple pendulum is  $\frac{d^2x}{dt^2} + W_0^2x = F_0 \sin t$ , where  $W_0$  and  $F_0$  are constants. Also initially  $x = 0, \frac{dx}{dt} = 0$  solve it. (07 Marks)

### Module-3

- 5 a. Find the PDE by eliminating the function from  $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$ . (06 Marks)
- b. Solve  $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$  given  $\frac{\partial z}{\partial y} = -2 \sin y$ , when  $x = 0$  and  $z = 0$ , when  $y$  is odd multiple of  $\frac{\pi}{2}$ . (07 Marks)
- c. Derive one-dimensional wave equation in usual notations. (07 Marks)



OR

- 6 a. Solve  $\frac{\partial^2 z}{\partial x^2} = a^2 z$  given that when  $x = 0$   $\frac{\partial z}{\partial x} = a \sin y$  and  $z = 0$ . (06 Marks)
- b. Solve  $x(y - z) p + y(z - x) q = z(x - y)$ . (07 Marks)
- c. Find all possible solution of  $U_t = C^2 U_{xx}$  one dimensional heat equation by variable separable method. (07 Marks)

Module-4

- 7 a. Test for convergence for  
 $1 + \frac{2!}{2^2} + \frac{3!}{3^2} + \frac{4!}{4^2} + \dots$  (06 Marks)
- b. Find the series solution of Legendre differential equation  $(1 - x^2)y'' - 2xy' + n(n + 1) = 0$  leading to  $P_n(x)$ . (07 Marks)
- c. Prove the orthogonality property of Bessel's function as  
 $\int_0^1 x \bar{j}_n(\alpha x) \bar{j}_n(\beta x) dx = 0 \quad \alpha \neq \beta$  (07 Marks)

OR

- 8 a. Test for convergence for  
 $\sum \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$  (06 Marks)
- b. Find the series solution of Bessel differential equation  $x^2 y'' + xy' + (n^2 - x^2)y = 0$  Leading to  $\bar{j}_n(x)$ . (07 Marks)
- c. Express the polynomial  $x^3 + 2x^2 - 4x + 5$  interms of Legendre polynomials. (07 Marks)

Module-5

- 9 a. Using Newton's forward difference formula find  $f(38)$ . (06 Marks)
- |      |     |     |     |     |     |     |
|------|-----|-----|-----|-----|-----|-----|
| x    | 40  | 50  | 60  | 70  | 80  | 90  |
| f(x) | 184 | 204 | 226 | 250 | 276 | 304 |
- b. Find the real root of the equation  $x \log_{10} x = 1.2$  by Regula falsi method between 2 and 3 (Three iterations). (07 Marks)
- c. Evaluate  $\int_4^{5.2} \log x dx$  by Weddle's rule considering six intervals. (07 Marks)

OR

- 10 a. Find  $f(9)$  from the data by Newton's divided difference formula:
- |      |     |     |      |      |      |
|------|-----|-----|------|------|------|
| x    | 5   | 7   | 11   | 13   | 17   |
| f(x) | 150 | 392 | 1452 | 2366 | 5202 |
- (06 Marks)
- b. Using Newton - Raphson method, find the real root of the equation  $x \sin x + \cos x = 0$  near  $x = \pi$ . (07 Marks)
- c. By using Simpson's  $\left(\frac{1}{3}\right)$  rule, evaluate  $\int_0^6 \frac{dx}{1+x^2}$  by considering seven ordinates. (07 Marks)

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