USN

14MAT11

First Semester B.E. Degree Examination, Dec.2019/Jan.2020

Engineering Mathematics - I

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. If $Y = a\cos(\log x) + b\sin(\log x)$, then show that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(07 Marks)**

b. Find the angle of intersection of the curves $r = a\cos\theta$ and $2r = a$. **(06 Marks)**

c. Derive an expression to find the radius of curvature in Polar form. **(07 Marks)**

OR

2 a. If $\sin^{-1} y = 2\log(x+1)$. Prove that $(x+1)^2y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$. **(07 Marks)**

b. Find the Pedal equation for $r = a\,\text{cosec}^2\,\theta/2$. **(06 Marks)**

c. Show that the radius of curvature at any point θ on the Cycloid $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$ is $4a\cos(\theta/2)$. **(07 Marks)**

Module-2

3 a. Using Maclaurin's series, expand $\log(\sec x)$ upto x^4. **(07 Marks)**

b. If $Z = e^{ax+by} f(ax - by)$, prove that
$b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$. **(06 Marks)**

c. If $u = f(x/y,\ y/z,\ z/x)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. **(07 Marks)**

OR

4 a. Evaluate $\lim_{x\to 0}\left(\frac{a^x + b^x + c^x}{3}\right)^{1/x}$. **(07 Marks)**

b. If $\cos u = \frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{-\cot u}{2}$. **(06 Marks)**

c. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. **(07 Marks)**

Module-3

5 a. A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$ and $z = 2t - 5$. Find the components of velocity and acceleration at $t = 1$ in the direction $\hat{i} - 2\hat{j} + 2\hat{k}$. **(07 Marks)**

b. Using differentiation under the integral sign, show that $\int_0^\pi \frac{\log(1+a\cos x)}{\cos x}dx = \pi\sin^{-1} a$. **(06 Marks)**

c. Use general rules to trace the curve $y^2(a-x) = x^3$, $a > 0$. **(07 Marks)**

OR

6 a. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) along $2i - j - 2k$. **(07 Marks)**

b. Show that div (Curl $\vec{A}$) = 0. **(06 Marks)**

c. Show that $\vec{F} = (y + z)i + (z + x)j + (x + y)k$ is irrotational. Also find ϕ such that $\vec{F} = \nabla \phi$. **(07 Marks)**

Module-4

7 a. Obtain the reduction formula for $\int \sin^m x \cos^n x \, dx$, where m and n are positive integers. **(07 Marks)**

b. Solve $(xy^3 + y)\,dx + 2(x^2y^2 + x + y^4)\,dy = 0$. **(06 Marks)**

c. Find the orthogonal trajectories of family of curves $r = 4a \sec\theta \tan\theta$. **(07 Marks)**

OR

8 a. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$. **(07 Marks)**

b. Solve $\frac{dy}{dx} - y\tan x = y^2 \sec x$. **(06 Marks)**

c. If the air is maintained at 30^0 C and the temperature of the body cools from 80^0C to 60^0C in 12 mins, find the temperature of the body after 24 mins. **(07 Marks)**

Module-5

9 a. Solve $2x + y + 4z = 12$, $4x + 11y - z = 33$, $8x - 3y + 2z = 20$ by Gauss elimination method. **(07 Marks)**

b. Diagonalize the Matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$. **(06 Marks)**

c. Determine the largest eigen value and the corresponding eigen vector of

$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ using Rayleigh's Power Method. **(07 Marks)**

OR

10 a. Solve by LU decomposition method $4x_1 + x_2 + x_3 = 4$, $x_1 + 4x_2 - 2x_3 = 4$, $3x_1 + 2x_2 - 4x_3 = 6$. **(07 Marks)**

b. Show that the transformation, $y_1 = 2x_1 - 2x_2 - x_3$, $y_2 = -4x_1 + 5x_2 + 3x_3$, $y_3 = x_1 - x_2 - x_3$ is regular and find the inverse transformation. **(06 Marks)**

c. Reduce the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 6x_1x_3 + 2x_2x_3$ into canonical form by orthogonal transformation. **(07 Marks)**

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