



Time: 3 hrs.

Seventh Semester B.E. Degree Examination, Dec.2019/Jan.2020
Digital Signal Processing

Max. Marks:100

- Note: 1. Answer any FIVE full questions, selecting at least TWO questions from each part.**
2. Use of Filter Table is permitted

PART – A

- 1 a. Find the N-point DFT of the sequence $x(n) = e^{j\omega n}$, $0 \leq n \leq N - 1$ and $\omega = \frac{2\pi}{N}$. (05 Marks)
- b. Find the 8-point DFT of the sequence $x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$ (10 Marks)
- c. Establish the relation between DFT and z-transform. (05 Marks)
- 2 a. State and prove circular folding property of an N-point sequence. (05 Marks)
- b. Find the four point circular convolution of the sequence $x(n) = \cos\left(\frac{n\pi}{2}\right)$, $h(n) = 2^n$; $0 \leq n \leq 3$. (05 Marks)
- c. Let $x(n)$ be a finite length sequence with $X(K) = (0, 1 + j, 1, 1 - j)$. Using the properties of DFT, find DFT of the following sequences.
 - i) $x_1(n) = e^{j\pi/2^n} x(n)$
 - ii) $x_2(n) = \cos\left(\frac{n\pi}{2}\right) x(n)$
 - iii) $x_3(n) = x((n-1))_4$
 - iv) $x_4(n) = (0, 0, 1, 0) \otimes_4 x(n)$ (10 Marks)
- 3 a. In the direct computation of 64-point DFT of $x(n)$ how many (i) complex multiplications (ii) complex additions (iii) Real multiplications (iv) Real additions (v) Trigonometric function evaluations are required? (05 Marks)
- b. Prove : i) Symmetry ii) Periodicity property of a twiddle factor. (05 Marks)
- c. Find the output $y(n)$ of a filter whose impulse response $h(n) = \left(\downarrow 3, 2, 1, 1\right)$ and input signal $x(n) = \{1, 2, 3, 3, 2, 1, -1, -2, -3, 5, 6, -1, 2, 0, 2, 1\}$ using overlap add method. Assume the length of the block as 7. (10 Marks)
- 4 a. Find 8-point DFT of a sequence $x(n) = \{0, 0.707, 1, 0.707, 0, -0.707, -1, -0.707\}$. Using radix -2 DITFFT algorithm. Draw the signal flow graph. (06 Marks)
- b. Develop 8-point DIFFFT radix-2 algorithm and draw the signal flow graph. (10 Marks)
- c. Given $x(n) = \{1, 0, 1, 0\}$. Find $X(2)$ using Goertzel algorithm. (04 Marks)

PART – B

- 5 a. For an analog Butterworth filter, derive an expression for order 'N' of lowpass filter. (05 Marks)
- b. Compare Butterworth and Chebychev filters. (05 Marks)

c. Design a Chebychev – I filter to meet the following specifications :

- i) Passband ripple : ≤ 2 dB
- ii) Passband edge frequency : 1 rad/sec
- iii) Stopband attenuation : ≥ 20 dB
- iv) Stopband edge frequency : 1.3 rad/sec.

(10 Marks)

6 a. A low pass filter is to be designed with the following desired frequency response

$$H_d(e^{j\omega}) = H_d(\omega) = \begin{cases} e^{-j2\omega}, & |\omega| < \frac{\pi}{4} \\ 0, & \frac{\pi}{4} < |\omega| < \pi \end{cases}$$

Determine the filter coefficients $h_d(n)$ and $h(n)$ if $w(n)$ is a rectangular window defined as

$$\text{follows : } W_R(n) = \begin{cases} 1, & 0 \leq n \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

Also find the frequency response, $H(\omega)$ of the resulting FIR filter.

(12 Marks)

b. Write equations of any four different windows used in design of FIR filters.

(08 Marks)

7 a. Derive the expression for the Bilinear transformation, to transform an analog filter to a digital filter, by trapezoidal rule and explain the mapping from s-plane to z-plane. (10 Marks)

b. Let $H_a(s) = \frac{b}{(s+a)^2 + b^2}$ be a causal second order analog transfer function. Show that the causal second order digital transfer function $H(z)$ obtained from $H_a(s)$ through impulse invariance method is given by $H(z) = \frac{e^{-aT} \sin bT z^{-1}}{1 - 2e^{-aT} \cos bT z^{-1} + e^{-2aT} z^{-2}}$. Also find $H(z)$ when

$$H_a(s) = \frac{1}{s^2 + 2s + 2}.$$

(10 Marks)

8 a. Realize the linear phase FIR filter with impulse response $h(n) = (\frac{1}{2})^n [u(n) - u(n-4)]$

(05 Marks)

b. Obtain the direct form – I, direct form – II cascade and parallel form realizations for the following system.

$$y(n] = x(n) + \frac{1}{4} x(n-1) - \frac{1}{8} y(n-3) - \frac{1}{2} y(n-2) - y(n-1)$$

(15 Marks)
