ACHARYA INSTITUTE OF TECHNOLOGY
Library
Date........
BANGALORE

USN

10CS34

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Define set, power set, complement of a set. Give one example each. **(06 Marks)**

b. Determine the sets A and B given that $A - B = \{1, 3, 5, 7, 9, 11\}$, $A \cap B = \{2, 8\}$ and $A \cup B = \{1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12\}$. **(04 Marks)**

c. Let X be the set of all three digit integers, that is $X = \{x \text{ is an integer} / 100 \le x \le 999\}$. If A_i is the set of integers from X in which i^{th} digit is i, compute the cardinality of the set $A_1 \cup A_2 \cup A_3$. **(05 Marks)**

d. Find probability of two persons A and B contradicting when they narrate same story, given that A speaks 60% true and B speaks 20% false. **(05 Marks)**

2 a. Define Tautology. Show that $\{[(p \vee q) \to r] \wedge \neg p\} \to (q \to r)$ is a tautology by constructing truth table. **(05 Marks)**

b. If q = 1, find all possible truth values which make the statement 'S' have truth value 1. $S : q \to [(\neg p \vee r) \wedge \neg s] \wedge [\neg s \to (\neg r \wedge q)]$. **(05 Marks)**

c. Verify the following statement without using truth table:
$[(p \to r) \wedge (\neg p \to q) \wedge (q \to s)] \to (\neg r \to s)$ **(05 Marks)**

d. Test the validity of the following argument :
If Ravi studies then he will pass DMS.
If Ravi does not play cricket then he will study.
Ravi failed in DMS.
Therefore Ravi played cricket. **(05 Marks)**

3 a. If $p(x) : x \ge 0$, q(x): x is even, and
r(x) : $x^2 - 3x - 4 = 0$. For $x \in R$, Find the truth values of,
(i) $(\exists x)[p(x) \wedge q(x)]$
(ii) $(\forall x)[q(x) \to r(x)]$ **(04 Marks)**

b. Define open statement. Write the negation of the statement " If all triangles are right angled then no triangle is equilateral". **(05 Marks)**

c. Test the validity of the following:

$$\begin{array}{l} (\forall x)[p(x) \vee q(x)] \\ (\forall x)[\neg q(x) \vee r(x)] \\ (\forall x)[s(x) \to \neg r(x)] \\ (\exists x)\neg p(x) \\ \hline \therefore (\exists x)\ \neg s(x) \end{array}$$

(06 Marks)

d. Give direct proof of the statement. "If m, $n \in z^+$ are perfect squares then product m.n is a perfect square". **(05 Marks)**

4 a. Prove by using principle of mathematical induction,
$1.3+2.4+3.5+....n(n+2)=\frac{n(n+1)(2n+7)}{6}$. **(05 Marks)**

b. A sequence $\{a_n\}$ is defined recursively by $a_n = a_{n-1} + n$ for $n \geq 2$, $a_1 = 7$. Find the explicit form of a_n. **(05 Marks)**

c. Let F_n denote the n^{th} Fibonacci number. Prove that $\sum_{i=1}^{n} F_i^2 = F_n \times F_{n+1}$. **(05 Marks)**

d. Prove that every positive integer $n \geq 24$ can be written as a sum of 5's and 7's. **(05 Marks)**

PART – B

5 a. Define Cartesian product of two sets. For any three non-empty sets A, B, C prove that $A\times(B-C)=(A\times B)-(A\times C)$. **(05 Marks)**

b. Check if each of the following functions, $f: Z \to z$ is one – one.
(i) $f(x)=2x+1$ (ii) $f(x)=x^3-x$ **(05 Marks)**

c. Let $f,g: R \to R$ be defined as $f(x)=2x+1$, $g(x) = x/3$. Compute g o f and show that g o f is invertible. What is $(g \text{ o } f)^{-1}(x) = ?$ **(05 Marks)**

d. A magnetic tape contains a collection of 5 lakh strings, made up of four or fewer number of English letters. Can all of these strings in the collection be distinct? Prove. **(05 Marks)**

6 a. For $A = \{1, 2, 3, 4\}$. Let a relation $R: A \to A$ be
$R = \{(1, 1)(1, 2)(1, 3)(2, 1)(2, 4)(3, 1)(3, 3)(4, 1)(4, 3)\}$
(i) Draw digraph of R.
(ii) Write the matrix M(R).
(iii) Find R^2 **(06 Marks)**

b. Draw Hasse diagram for the poset represented by the positive divisors of 72. **(06 Marks)**

c. Define equivalence relation. Let $A=\{1, 2, 3, 4, 5\}$ Let $R : A\times A$, $(x_1\ y_1)\,R\,(x_2\ y_2)$ iff $x_1+y_1=x_2+y_2$. Verify if R is an equivalence relation. Find the partition of A induced by R. **(08 Marks)**

7 a. State and prove Lagrange's theorem. **(06 Marks)**

b. Let $S=R\times R$. Define an operation * on S as $(u,v)*(x,y)=(ux, vx+y)$. Prove that (s, *) is a non-abelian group. **(06 Marks)**

c. Let $f: G_1 \to G_2$ be a homomorphism. Show that identity e_1 of G_1 is mapped to identity e_2 of G_2 under f. **(04 Marks)**

d. Define cyclic group. Show that $S=\{1,-1,i,-i\}$ is a cyclic group under multiplication ($i=\sqrt{-1}$). **(04 Marks)**

8 a. Show that Z_5 is an integral domain. **(06 Marks)**

b. In a group code, prove that minimum distance between distinct code words is the minimum weight of the non-zero elements of the code. **(06 Marks)**

c. The generator matrix for an encoding function $E: Z_2^3 \to Z_2^6$ is given by the matrix G.

$$G=\begin{bmatrix}1&0&0&1&1&0\\0&1&0&0&1&1\\0&0&1&1&0&1\end{bmatrix}$$

. Find the code words assigned to 110, 010. Also find associated parity check matrix and decode the generated code words. **(08 Marks)**

* * * * *

VTU 25-01-2020 01:38:03pm AY - AY - AY